

Math 233 - Quiz 1

January 19, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) Let \vec{u} be the vector from $P(3, -5)$ to $Q(-2, 8)$.

(a) Find the component form of \vec{u} .

$$\vec{u} = \vec{PQ} = (-2-3)\hat{i} + (8-(-5))\hat{j} = -5\hat{i} + 13\hat{j} = \langle -5, 13 \rangle$$

(b) Compute $\|\vec{u}\|$.

$$\|\vec{u}\| = \sqrt{(-5)^2 + (13)^2} = \sqrt{25+169} = \sqrt{194}$$

(c) Find a unit vector whose direction is opposite that of \vec{u} .

$$\frac{-\vec{u}}{\|\vec{u}\|} = -\frac{1}{\sqrt{194}}(-5\hat{i} + 13\hat{j}) = \frac{5}{\sqrt{194}}\hat{i} - \frac{13}{\sqrt{194}}\hat{j} = \left\langle \frac{5}{\sqrt{194}}, \frac{-13}{\sqrt{194}} \right\rangle$$

(d) Compute $\|2\vec{u}\|$.

$$\|2\vec{u}\| = 2\|\vec{u}\| = 2\sqrt{194}$$

2. (2 points) Find a vector of length 5 that has the direction of $\vec{w} = \langle 2, -3 \rangle$.

$$\|\vec{w}\| = \sqrt{4+9} = \sqrt{13}$$

$$\frac{5\vec{w}}{\|\vec{w}\|} = \frac{10}{\sqrt{13}}\hat{i} - \frac{15}{\sqrt{13}}\hat{j} = \left\langle \frac{10}{\sqrt{13}}, \frac{-15}{\sqrt{13}} \right\rangle$$

3. (3 points) Let $\vec{x} = \langle 1, 2 \rangle$ and $\vec{y} = \langle -3, 7 \rangle$. Compute $\|2\vec{x} - 4\vec{y}\|$.

$$2\vec{x} - 4\vec{y} = \langle 2, 4 \rangle - \langle -12, 28 \rangle = \langle 14, -24 \rangle$$

$$\|2\vec{x} - 4\vec{y}\| = \sqrt{(14)^2 + (-24)^2} = \sqrt{196 + 576} = \sqrt{772}$$

4. (1 point) The vector $\vec{v} = \langle -2, -4 \rangle$ has initial point $(-1, 7)$. Find its terminal point.

$$P(-1, 7) \quad Q(x, y)$$

$$Q(-3, 3)$$

$$\vec{v} = \vec{PQ} = \langle -2, -4 \rangle = \langle x+1, y-7 \rangle$$

$$x+1 = -2 \Rightarrow x = -3$$

$$y-7 = -4 \Rightarrow y = 3$$