Math 233 - Quiz 2

January 26, 2023

Name _____ key _____

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (4 points) In each part of this problem, you are given information about $\vec{x}$ and $\vec{y}$. Determine whether the angle between $\vec{x}$ and $\vec{y}$ is acute, right, or obtuse. Explain as necessary.

    (a) $\vec{x} \cdot \vec{y} = -17$ $\implies$ $\cos\theta$ IS NEG $\implies$ $\boxed{\theta \text{ IS OBTUSE}}$

    (b) $\vec{x} = 3\hat{\imath} - 7\hat{\jmath} + 5\hat{k}$ and $\vec{y} = 4\hat{\imath} + \hat{\jmath} - \hat{k}$

    $\vec{X} \cdot \vec{y} = 12 - 7 - 5 = 0$ $\implies$ $\boxed{\theta \text{ IS A RIGHT } \angle}$

    (c) $\dfrac{\vec{x} \cdot \vec{y}}{\|\vec{x}\| \|\vec{y}\|} = \dfrac{\sqrt{3}}{2}$ $\implies$ $\cos\theta$ IS POS $\implies$ $\boxed{\theta \text{ IS ACUTE}}$

    IN FACT, $\theta = 30°$.

2. (3 points) Determine the measure of the angle between the vectors $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} - 9\hat{k}$ and $\vec{b} = 2\hat{\jmath} + 8\hat{k}$. Write your final answer in degrees, rounded to the nearest hundredth.

$\vec{a} \cdot \vec{b} = 3(0) + 4(2) - 9(8) = -64$

$\cos\theta = \dfrac{-64}{\sqrt{106}\sqrt{68}} \approx -0.7538$

$\|\vec{a}\| = \sqrt{9 + 16 + 81} = \sqrt{106}$ $\|\vec{b}\| = \sqrt{0 + 4 + 64}$
$= \sqrt{68}$

$\boxed{\theta \approx 138.92°}$

3. (3 points) Show that these points are collinear.

$$P(3, -2, 1), \quad Q(0, 7, -14), \quad R(5, -8, 11)$$

$\vec{PQ} = -3\hat{\imath} + 9\hat{\jmath} - 15\hat{k}$

$\vec{PR} = 2\hat{\imath} - 6\hat{\jmath} + 10\hat{k}$

$\implies \boxed{\vec{PQ} = -\dfrac{3}{2}\vec{PR}}$

$\implies \vec{PQ}$ AND $\vec{PR}$ ARE PARALLEL

$\implies P, Q, R$ COLLINEAR.