Math 233 - Quiz 3

Name __key__

February 2, 2023

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find a unit vector that is orthogonal to both $\vec{u} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{v} = -4\hat{\imath} + 2\hat{\jmath} + \hat{k}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ -4 & 2 & 1 \end{vmatrix}$$

$$= \hat{\imath}(-2-6) - \hat{\jmath}(1+12) + \hat{k}(2-8)$$

$$= -8\hat{\imath} - 13\hat{\jmath} - 6\hat{k}$$

I'll use $-\vec{u} \times \vec{v} = 8\hat{\imath} + 13\hat{\jmath} + 6\hat{k}$

$$\| -\vec{u} \times \vec{v} \| = \sqrt{64 + 169 + 36} = \sqrt{269}$$

$$\frac{-\vec{u} \times \vec{v}}{\| \vec{u} \times \vec{v} \|} = \boxed{\frac{1}{\sqrt{269}} \left(8\hat{\imath} + 13\hat{\jmath} + 6\hat{k} \right)}$$

2. (3 points) Find a set of parametric equations for the line segment from $P(1, -4, 3)$ to $Q(6, 5, 1)$.

$$\vec{PQ} = (6-1)\hat{\imath} + (5+4)\hat{\jmath} + (1-3)\hat{k}$$

$$= 5\hat{\imath} + 9\hat{\jmath} - 2\hat{k}$$

Starting from $P(1, -4, 3)$

$$\boxed{\begin{array}{l} x = 1 + 5t \\ y = -4 + 9t \qquad 0 \le t \le 1 \\ z = 3 - 2t \end{array}}$$

3. (2 points) A line is described by the symmetric equations $\dfrac{x+2}{5} = \dfrac{7-y}{2} = z - 8$. Find a point on the line, and find a vector parallel to the line.

$$\frac{x+2}{5} = \frac{y-7}{-2} = \frac{z-8}{1}$$

$$\boxed{\begin{array}{l} \text{Point: } (-2, 7, 8) \\ \text{Parallel vector: } 5\hat{\imath} - 2\hat{\jmath} + \hat{k} \end{array}}$$

4. (2 points) Find a unit vector that is normal to the plane given by the $2x + y - 3z = 8$.

$$\vec{n} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$$

$$\| \vec{n} \| = \sqrt{4 + 1 + 9} = \sqrt{14}$$

$$\frac{\vec{n}}{\| \vec{n} \|} = \boxed{\frac{2\hat{\imath} + \hat{\jmath} - 3\hat{k}}{\sqrt{14}}}$$