Name __key__

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find the function $\vec{r}(t)$ that satisfies

$$\vec{r}'(t) = e^{-2t}\,\hat{\imath} - t\,\hat{\jmath} + \frac{5t}{t^2+1}\,\hat{k}; \qquad \vec{r}(0) = 5\hat{\imath} + 3\hat{\jmath} - 2\hat{k}.$$

$\hat{\imath})$ $\displaystyle\int e^{-2t}\,dt$

$\qquad = -\frac{1}{2}e^{-2t} + c_1$

$\hat{\jmath})$ $\displaystyle\int -t\,dt = -\frac{1}{2}t^2 + c_2$

$\hat{k})$ $\displaystyle\int \frac{5t}{t^2+1}\,dt = \int \frac{5}{2}\frac{1}{u}\,du$

$\quad u = t^2+1$
$\quad du = 2t\,dt$
$\qquad = \frac{5}{2}\ln(t^2+1) + c_3$

$\vec{r}(0) = 5\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$5 = -\frac{1}{2} + c_1 \;\Rightarrow\; c_1 = \frac{11}{2}$

$3 = c_2$

$-2 = \underset{0}{\underline{\frac{5}{2}\ln(1)}} + c_3 \;\Rightarrow\; c_3 = -2$

$$\vec{r}(t) = \left(-\frac{1}{2}e^{-2t} + \frac{11}{2}\right)\hat{\imath} + \left(-\frac{1}{2}t^2 + 3\right)\hat{\jmath} + \left(\frac{5}{2}\ln(t^2+1) - 2\right)\hat{k}$$

2. (3 points) Let $\vec{r}(t) = \sin 6t\,\hat{\imath} + \cos 6t\,\hat{\jmath} + 8t\,\hat{k}$. Compute $\hat{T}(t)$.

$\vec{r}'(t) = 6\cos 6t\,\hat{\imath} - 6\sin 6t\,\hat{\jmath} + 8\hat{k}$

$\|\vec{r}'(t)\| = \sqrt{36\cos^2 6t + 36\sin^2 6t + 64}$

$\qquad = \sqrt{36+64} = 10$

$$\hat{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{3\cos 6t\,\hat{\imath} - 3\sin 6t\,\hat{\jmath} + 4\hat{k}}{5}$$

3. (2 points) What are the three important characteristics of the principal unit tangent vector.

$\dfrac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \hat{T}(t)$

$\hat{T}(t)$ is

① TANGENT TO THE GRAPH OF $\vec{r}(t)$ AT t,

② A UNIT VECTOR, AND

③ POINTING IN THE DIRECTION OF MOTION ALONG THE GRAPH OF $\vec{r}$.