Math 233 - Quiz 5

February 23, 2023

Name **key**

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Let $\vec{r}(t) = 2t\,\hat{\imath} + (t+3)\,\hat{\jmath} + 6t\,\hat{k}$. Reparameterize $\vec{r}(t)$ in terms of the arc-length parameter.

$$\vec{r}'(t) = 2\hat{\imath} + \hat{\jmath} + 6\hat{k}$$

$$\|\vec{r}'(t)\| = \sqrt{4+1+36}$$

$$= \sqrt{41}$$

Using $t_0 = 0$

$$s = \int_0^t \sqrt{41}\, d\tau = \sqrt{41}\, t \quad \Rightarrow \quad t = \frac{s}{\sqrt{41}}$$

$$\boxed{\vec{R}(s) = \frac{2s}{\sqrt{41}}\,\hat{\imath} + \left(\frac{s}{\sqrt{41}}+3\right)\hat{\jmath} + \frac{6s}{\sqrt{41}}\,\hat{k}}$$

2. (3 points) Consider the curve described by $\vec{r}(t) = e^t\,\hat{\imath} + 2t\,\hat{\jmath} - 7\hat{k}$. Compute the curvature at the point where $t = 0$.

$$\vec{r}'(t) = e^t\,\hat{\imath} + 2\hat{\jmath}$$

$$\vec{r}''(t) = e^t\,\hat{\imath}$$

$$\vec{r}' \times \vec{r}'' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ e^t & 2 & 0 \\ e^t & 0 & 0 \end{vmatrix} = -2e^t\,\hat{k}$$

$$\|\vec{r}'(t)\| = \sqrt{e^{2t}+4}$$

$$K(t) = \frac{2e^t}{(e^{2t}+4)^{3/2}}$$

$$\boxed{K(0) = \frac{2}{5^{3/2}}}$$

3. (3 points) Let $\vec{r}(t) = \sin(2t)\,\hat{\imath} + t\,\hat{\jmath} + \cos(2t)\,\hat{k}$. Compute $\hat{N}(t)$.

$$\vec{r}'(t) = 2\cos 2t\,\hat{\imath} + \hat{\jmath} - 2\sin 2t\,\hat{k}$$

$$\|\vec{r}'(t)\| = \sqrt{4+1} = \sqrt{5}$$

$$\hat{T}(t) = \frac{2\cos 2t\,\hat{\imath} + \hat{\jmath} - 2\sin 2t\,\hat{k}}{\sqrt{5}}$$

$$\hat{T}'(t) = \frac{-4\sin 2t\,\hat{\imath} - 4\cos 2t\,\hat{k}}{\sqrt{5}}$$

$$\boxed{\hat{N}(t) = -\sin 2t\,\hat{\imath} - \cos 2t\,\hat{k}}$$

4. (1 point) Refer to problem 1. Explain why $\hat{N}(t)$ does not exist for $\vec{r}(t)$.

THE MOTION IS ALONG A LINE. $\hat{T}(t)$ IS CONSTANT.

$$\therefore \quad \hat{T}'(t) = \vec{0}$$