

Math 233 - Quiz 6

March 2, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (8 points) A football is kicked off the ground at an angle of 30° above the horizontal and with an initial speed of 65 ft/sec. (Ignore air resistance in this problem and use $g = 32 \text{ ft/s}^2$.)

(a) Find the vector-valued function that gives the position of the football at time t .

$$\vec{r}(t) = 65 \cos 30^\circ t \hat{i} + (-16t^2 + 65 \sin 30^\circ t) \hat{j}$$

$$\vec{r}(t) = \frac{65\sqrt{3}}{2} t \hat{i} + (-16t^2 + \frac{65}{2} t) \hat{j}$$

(b) What horizontal distance does the football travel while it is in the air?

$$-16t^2 + \frac{65}{2} t = 0$$

$$t(-16t + \frac{65}{2}) = 0$$

$$t = 0 \text{ or } t = \frac{65}{32} \text{ s}$$

$$\frac{65\sqrt{3}}{2} \cdot \frac{65}{32} = \frac{4225\sqrt{3}}{64} \approx 114.34 \text{ FT}$$

(c) Set up the definite integral that gives the length of the path of the football. Use your calculator to estimate the value of the integral.

$$\vec{r}'(t) = \frac{65\sqrt{3}}{2} \hat{i} + (-32t + \frac{65}{2}) \hat{j}$$

$$\text{LENGTH} = \int_0^{65/32} \sqrt{\left(\frac{65\sqrt{3}}{2}\right)^2 + \left(-32t + \frac{65}{2}\right)^2} dt \approx 120.41 \text{ FT}$$

(d) When does the football reach its maximum height?

$$-32t + \frac{65}{2} = 0 \Rightarrow t = \frac{65}{64} = 1.015625 \text{ SEC}$$

2. (1 point) What can we say about the motion of a moving object if the normal component of its acceleration is zero?

THE DIRECTION IS NOT CHANGING. MOTION IS ALONG A LINE.

3. (1 point) What can we say about the motion of a moving object if the tangential component of its acceleration is zero?

THE SPEED IS NOT CHANGING. ONLY THE DIRECTION OF MOTION IS CHANGING.