

# Math 233 - Quiz 8

March 30, 2023

Name key

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find the linearization of  $f(x, y) = \frac{1}{1+x-y}$  at the point where  $(x, y) = (2, 1)$ ,

$$f(2, 1) = \frac{1}{2} \quad f_y(x, y) = \frac{1}{(1+x-y)^2}$$

$$f_x(x, y) = \frac{-1}{(1+x-y)^2} \quad f_y(2, 1) = \frac{1}{4}$$

$$f_x(2, 1) = -\frac{1}{4}$$

$$L(x, y) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{4}(y-1)$$

2. (3 points) The volume of a right circular cylinder of radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ . Use differentials to approximate  $\Delta V$  when  $r$  changes from 1 to 0.98 and  $h$  changes from 5 to 5.04.

$$\Delta r = -0.02 \quad \Delta h = 0.04$$

$$dV = 2\pi r h dr + \pi r^2 dh$$

$$\Delta V \approx 2\pi r h \Delta r + \pi r^2 \Delta h$$

$$\Delta V \approx 2\pi(1)(5)(-0.02) + \pi(1)^2(0.04)$$

$$= -0.2\pi + 0.04\pi = -0.16\pi \approx -0.503$$

3. (4 points) Use the definition of differentiability to show that  $f(x, y) = xy - x$  is differentiable everywhere.

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x, y) = [(x+\Delta x)(y+\Delta y) - (x+\Delta x)] - [xy - x]$$

$$= \cancel{xy} + y\Delta x + \underline{x\Delta y} + \underline{\Delta x\Delta y} - \cancel{x} - \cancel{\Delta x} - \cancel{xy} + \cancel{x}$$

$$= \underline{(y-1)\Delta x} + \underline{x\Delta y} + \Delta y \Delta x + 0 \Delta y$$

$\uparrow \epsilon_1$                        $\uparrow \epsilon_2$

$$= f_x(x, y) \Delta x + f_y(x, y) \Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

WHERE  $\epsilon_1 = \Delta y$  AND  $\epsilon_2 = 0$

AND  $\epsilon_1 \rightarrow 0$  AND  $\epsilon_2 \rightarrow 0$  AS  $(\Delta x, \Delta y) \rightarrow (0, 0)$ .

ALL OF THIS IS TRUE FOR ALL  $(x, y) \in \mathbb{R}^2$ .

↓  
f IS DIFF. ON  $\mathbb{R}^2$ .