Math 233 - Quiz 9

April 6, 2023

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Find an equation of the plane tangent to the graph of $x^2 + 4xy - y^3 - z = 0$ at the point where $(x, y) = (3, 2)$.

$z = x^2 + 4xy - y^3$

$z(3,2) = 25$

$z_x(x,y) = 2x + 4y$

$z_x(3,2) = 14$

$z_y(x,y) = 4x - 3y^2$

$z_y(3,2) = 0$

TANGENT PLANE:

$z = 25 + 14(x-3) + 0(y-2)$

OR

$\boxed{14x - z = 17}$

2. (4 points) Let $z = \ln(x^2 + 4y)$, where $x = r\cos\theta$ and $y = r\sin\theta$. Use the appropriate multi-variable chain rule to determine formulas for $\partial z/\partial r$ and $\partial z/\partial\theta$.

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \frac{2x}{x^2+4y}\cdot\cos\theta + \frac{4}{x^2+4y}\cdot\sin\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = \frac{2x}{x^2+4y}\cdot(-r\sin\theta) + \frac{4}{x^2+4y}\cdot r\cos\theta$$

3. (3 points) The graph of the equation $x^3 + xy^2 = 3x^2 - y^2$ is called a *trisectrix of Maclaurin*. Use partial derivatives to find dy/dx.

$F(x,y) = x^3 + xy^2 - 3x^2 + y^2$

$$\frac{dy}{dx} = \frac{-F_x}{F_y} = \boxed{\frac{-(3x^2 + y^2 - 6x)}{2xy + 2y}}$$