Math 233 - Test 1

February 9, 2023

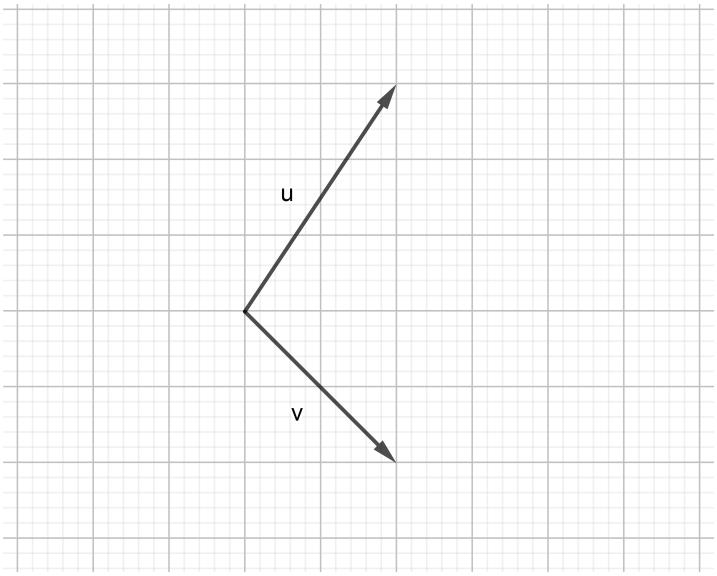
Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (8 points) The vector $\vec{w}$ lies in the xy-plane, has magnitude 8, and makes a 120° angle with the positive x-axis. Find the projection of $\vec{w}$ onto $\vec{v} = -10\hat{\imath} + 2\hat{\jmath}$.

2. (6 points) The figure below shows the vectors $\vec{u}$ and $\vec{v}$. Sketch and label the vectors $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, and $-\frac{3}{2}\vec{v}$.



3. (6 points) Show that the points $R\,(14, 39, 3)$, $S\,(5, -3, 0)$, and $T\,(-1, -31, -2)$ are collinear.

4. (6 points) Determine the measure of the angle that $\vec{w} = \sqrt{2}\,\hat{\imath} + 3\hat{\jmath} - \sqrt{7}\,\hat{k}$ makes with the positive y-axis. Write your answer in degrees.

5. (9 points) Let $\vec{v} = \hat{\imath} + \hat{\jmath} - 7\hat{k}$ and $\vec{w} = -5\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$. Show that $\vec{v} \times \vec{w}$ is orthogonal to $2\vec{v} + \vec{w}$.

6. (9 points) Consider the line in space that passes through the points $P\,(5,-2,1)$ and $Q\,(-6,3,-5)$.

   (a) Find symmetric equations for the line.

   (b) Find a set of parametric equations for the segment $\overline{PQ}$.

   (c) Find the midpoint of the segment $\overline{PQ}$. Referring to part (b), what value of your parameter coincides the midpoint?

7. (10 points) Find an equation of the plane that contains the point $P\,(2,4,-1)$ and all points on the line ℓ. Symmetric equations for ℓ are shown below.

$$\text{Line } \ell: \quad \frac{x-1}{2} = y+4 = \frac{z-5}{2}$$

8. (10 points) If P is a point on the line that has the direction of $\vec{v}$, then the distance from the line to a point Q is given by

$$D = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|}.$$

Find the distance from the origin to the line with parametric equations

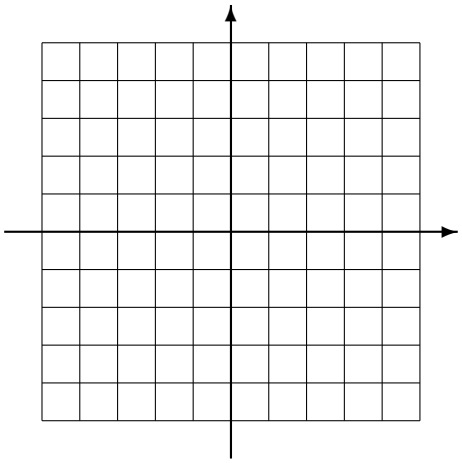
$$x = 1 + t, \quad y = 3 + t, \quad z = 5 + 4t.$$

9. (6 points) Determine the measure of the angle between the planes. Write your answer in degrees rounded to the nearest integer.

$$x - 3y + 6z = 4, \qquad 5x + y - z = 4$$

10. (8 points) Consider the vector-valued function $\vec{r}(t) = 2t\,\hat{\imath} - t^2\,\hat{\jmath}$.

(a) Sketch the graph of $\vec{r}(t)$. Show or describe the orientation of the curve.



(b) Compute $\|\vec{r}(t)\|$.

(c) Find a (nonzero) vector-valued function that is orthogonal to $\vec{r}(t)$ for every real number t.

(d) Describe the graph of the vector-valued function $\vec{r}(t) = 2t\,\hat{\imath} - t^2\,\hat{\jmath} + t\,\hat{k}$.

11. (8 points) Let $\vec{r}(t) = \dfrac{e^t - 1}{t}\,\hat{\imath} + \dfrac{\sin t}{t}\,\hat{\jmath} + \sin(\pi t)\,\hat{k}$.

(a) Determine the domain of $\vec{r}$.

(b) Compute $\vec{r}(4)$.

(c) Compute $\lim_{t \to 1} \vec{r}(t)$.

(d) Compute $\lim_{t \to 0} \vec{r}(t)$.

The following problem makes up the take-home portion of the test. This portion of the test is due February 14, 2023. You must work on your own.

12. (14 points) Consider the following planes

$$P_1 : 2x - 3y + 8z = 10, \qquad P_2 : x + 2y + 4z = 4.$$

(a) Show that the planes are not parallel.

(b) Find a set of parametric equations for the line of intersection of the planes.

(c) Find the distance from the point $R\,(9, 1, -3)$ to the plane P_1.

(d) Find symmetric equations for the line through $(2, 3, 4)$ and normal to the plane P_2.