Math 233 - Test 2

March 9, 2023

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) For $t \geq 0$, let $\vec{r}(t) = (\cos t + t\,\sin t)\,\hat{\imath} + (\sin t - t\,\cos t)\,\hat{\jmath}$. Compute the principal unit tangent vector, $\hat{T}(t)$.

2. (8 points) The velocity vector of a moving particle is given by

$$\vec{v}(t) = (\cos t)\,\hat{\imath} + (5\sin t)\,\hat{\jmath} + e^{-t}\,\hat{k}.$$

Find the position vector if the particle's motion began (at $t = 0$) at the point $(2, 7, 4)$.

3. (8 points) A curve in the xy-plane is described by the following parametric equations. Find the curvature function, $\kappa(t)$.

$$x = \frac{t^3}{3}, \quad y = \frac{t^2}{2}$$

4. (12 points) Let $\vec{r}(t) = t\,\hat{\imath} - \sin 4t\,\hat{\jmath} - \cos 4t\,\hat{k}$. Starting from $t = 0$, find the arc-length parameter, $s(t)$, and then reparameterize $\vec{r}$ in terms of s.

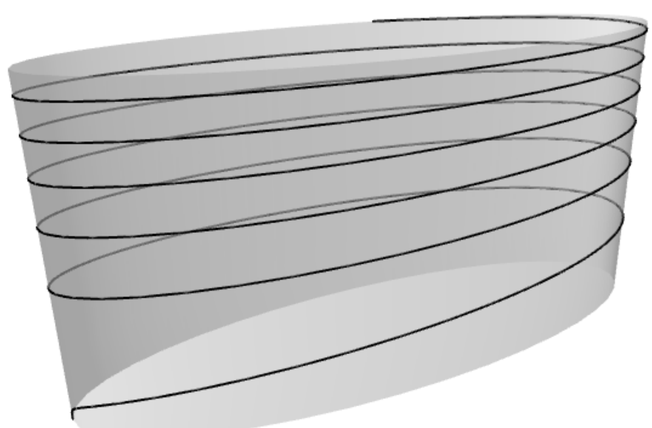
Follow-up: Show that when the function is reparameterized, its derivative has magnitude 1.

5. (10 points) Let $\vec{r}(t) = (2t+3)\,\hat{\imath} + (t^2-1)\,\hat{\jmath}$. Compute the tangential and normal components of acceleration.

6. (8 points) A wire is wrapped around an elliptical steel tube so that the wire has the shape of the graph of

$$\vec{r}(t) = 6\cos(t)\,\hat{\imath} + 2\sin(t)\,\hat{\jmath} + \sqrt{t}\,\hat{k}, \quad 0 \le t \le 36,$$

where $\vec{r}$ is in centimeters. Set up the definite integral that gives the length of the wire. Use your calculator to approximate the value of your integral.



7. (8 points) A baseball, hit 3 feet above the ground, leaves the bat at an angle of 45° and is caught by an outfielder at a height of 3 feet above the ground and 300 feet from home plate. What is the initial speed of the ball? (To receive full credit, you must write and use the vector-valued function $\vec{r}(t)$ that gives the position of the ball at time t. Also ignore air resistance and use $g \approx 32\,\mathrm{ft\,s^{-2}}$.)

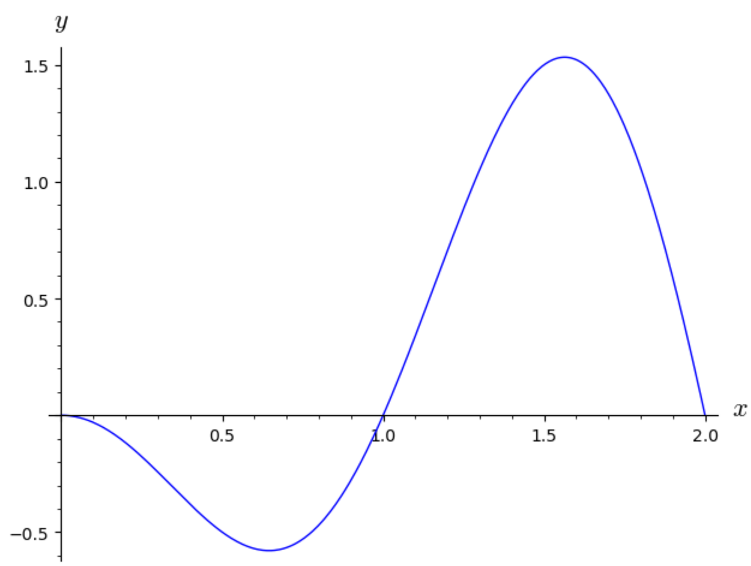
8. (6 points) Suppose a particle moves along the given curve from **right to left**. Sketch and label each of the following. Make note of the scale.

   (a) The principal unit tangent vector at the point of greatest curvature.

   (b) A point where the principal unit normal vector does not exist.

   (c) The principal unit normal vector at the point where $x = 0.7$.

   (d) (1 pt extra credit) The circle of curvature at the point where the radius of curvature is least.



9. (10 points) Consider the function $F(x, y) = \sqrt{1 + x^2 - |y|}$.

   (a) What is the domain of F?

   (b) Sketch the domain in the xy-plane.

   (c) Sketch the level curve $F(x, y) = 1$.

   (d) What is the range of F?

   (e) Compute $F(4, -8)$.

10. (8 points) Identify the graph of the **surface in space** described by each equation.

   (a) $\dfrac{y}{5} = 8x^2 + 7z^2$

   (b) $x^2 - \sqrt{8} = y^2 + z^2$

   (c) $\dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$

   (d) $x^2 - y^2 + z^2 = 0$

11. (6 points) Let $G(x, y, z) = x^2 + y^2 + z^2$.

(a) Compute $G(-1, 2, -3)$.

(b) What are the domain and range of G?

(c) Describe, in detail, the level surface $G(x, y, z) = 9$.

12. (10 points) Compute each limit.

(a) $\displaystyle\lim_{(x,y)\to(3,3)} \frac{(2x + y)^2 - (5y^2 + 4xy)}{x - y}$

(b) $\displaystyle\lim_{(x,y)\to(2,1)} \frac{xy^2 - 2y^2}{\sqrt{x} - \sqrt{2}}$