Math 233 - Test 3

April 13, 2023

Name ______________________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (14 points) Use the two-path test to show that each limit fails to exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy + y^3}{x^2 + y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(1,0)} \frac{(x-1)y^2}{(x-1)^3 + y^4}$

2. (2 points) If f is continuous at (x_0, y_0), then $\displaystyle\lim_{(x,y)\to(x_0,y_0)} f(x,y) =$ ____________________.

3. (8 points) Let $f(x, y) = \dfrac{xy}{x - y}$. Evaluate f_x and f_y at the point $(2, -2)$.

4. (8 points) Show that $z = e^x \sin y$ satisfies the equation $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

5. (2 points) Suppose f is defined on an open region in $\mathbb{R}^2$. We would expect $f_{xy} = f_{yx}$ as long as f_{xy} and f_{yx} are ____________________.

6. (8 points) Let $z = \sqrt{7 - x^2 + y^3}$. Use differentials to estimate the change in z as (x, y) moves from $(2, 1)$ to $(2.01, 0.97)$.

7. (8 points) Find the linearization of $f(x, y, z) = \tan^{-1}(x^2+6y+4z)$ at $(x, y, z) = (1, 0, 0)$. Then use your linearization to approximate $f(0.9, 0.1, 0.1)$.

8. (8 points) Suppose θ is implicitly defined as a function of x and y by the equation $y - x\tan\theta = 0$. Determine $\partial\theta/\partial x$ and $\partial\theta/\partial y$.

9. (8 points) Let $w = xy\cos z$, where $x = t$, $y = t^2$, and $z = \sin^{-1} t$. Use the appropriate chain rule to find a formula for dw/dt.

10. (2 points) At any point where f is differentiable, the directional derivative is greatest in the direction of the ___________________ ___________________.

11. (8 points) Let $f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$. Determine a unit vector in the direction of the maximum increase from the point $(5, -5, 5)$.

12. (8 points) Let $g(x, y) = x^2 + 2xy - 4y^2 + 4x - 6y + 4$. Determine all points for which $\nabla g(x, y) = \vec{0}$.

13. (2 points) Suppose $f(x, y)$ is a differentiable on $\mathbb{R}^2$. $\nabla f(x_0, y_0)$ is normal to the ____________________ ____________________ passing through (x_0, y_0).

14. (14 points) Consider the surface described by the equation $4x^2 - 2y^2 + z^2 = 12$.

(a) Identify the surface.

(b) Show that the point $(2, 2, 2)$ is on the surface.

(c) Find a vector normal to the surface at the point $(2, 2, 2)$.

(d) Find an equation of the plane tangent to the surface at $(2, 2, 2)$.

(e) Find a set of parametric equations for the line normal to the surface at $(2, 2, 2)$.