Math 233 - Final Exam A

Name ____________________________

May 9, 2023

Score ___________

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due May 11. You must work individually.

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1. (10 points) Let C be the closed curve in the plane made up of three line segments: from $(0,0)$ to $(1,0)$, from $(1,0)$ to $(0,1)$, and from $(0,1)$ to $(0,0)$. According to Green's theorem,

$$\int_C x^2y^3\,dx + x^2y\,dy = \iint_T (2xy - 3x^2y^2)\,dA,$$

where T is the triangular region inside the closed curve C. Evaluate the double integral.

2. (10 points) Let $\vec{F}(x,y,z) = z^2\,\hat{\imath} + 2y\,\hat{\jmath} + 2xz\,\hat{k}$. Consider the line integral

$$\int_C \vec{F}(x,y,z) \cdot d\vec{r}.$$

(a) Evaluate the line integral when C is the line segment from $(0,0,0)$ to $(1,1,2)$.

(b) Evaluate the line integral when C is the graph of $\vec{r}(t) = t\,\hat{\imath} + t^2\hat{\jmath} + 2t^3\,\hat{k}$ from $(0,0,0)$ to $(1,1,2)$.

(c) Let $f(x,y,z) = xz^2 + y^2 + 3$. Show that $\nabla f(x,y,z) = \vec{F}(x,y,z)$.

(d) Using the function f from part (c), compute $f(1,1,2) - f(0,0,0)$.