Math 233 - Final Exam B

Name _____

May 11, 2023

Score _____

Show all work to receive full credit. Supply explanations where necessary.

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1. (10 points) Two planes are described by the equations below.

$$P_1 : 3x - 4y + 5z = 8 \qquad P_2 : 7(x-4) + 3y - 3(z-9) = 0$$

Show that the planes are not parallel, and then find the measure of the angle between them. Write your final answer in degrees, rounded to the nearest hundredth.

2. (10 points) Find the arc-length parameterization (starting from $t = 0$) for the curve described by the vector-valued function

$$\vec{r}(t) = 5\cos t\,\hat{\imath} + 12t\,\hat{\jmath} + 5\sin t\,\hat{k}.$$

3. (10 points) A small projectile is launched from ground level with an initial speed of 98 m/s. Find the possible launch angles so that the range of the projectile is 490 m. (You may need to use a double angle trig formula.)

4. (10 points) Consider the surface described by the equation

$$x \ln(z^2) + 5y^2 e^{2x} = 42 + yz \cos(4x).$$

(a) Find an equation of the plane tangent to the surface at the point $(0, 3, 1)$.

(b) Find a set of parametric equations for the line normal to the surface at the point $(0, 3, 1)$.

5. (10 points) Find the critical points of $f(x, y) = x^3 + y^3 - 3x^2 - 3y^2 - 9x$. Then use the second partials test to classify the critical points and find the extreme values.

6. (10 points) Sketch the region of integration, reverse the order of integration, and evaluate.

$$\int_0^2 \int_{x^2}^4 xe^{y^2}\, dy\, dx$$

7. (10 points) The volume of a solid is given by

$$\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2}\, dy\, dx.$$

Sketch the region of integration, and then evaluate the integral by first converting to polar coordiantes.

8. (10 points) A region in space lies in the first octant (where $x, y, z \geq 0$) inside the cylinders $y = x^2$ and $y = 2 - x^2$, above the plane $z = 0$, and below the plane $z = 2 + y$. The volume of the region is $4\,\text{units}^3$. Use a triple integral to find the average value of $f(x, y, z) = x^2$ over the region.