Math 233 - Assignment 11

Name _____

April 25, 2024

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due May 2.

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1. Find the area of the upper half of the cardioid $r = 1 + \cos\theta$.

2. Convert to polar coordinates and evaluate.

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (x^2 + y^2)^2 \, dx \, dy$$

3. Evaluate by converting to polar coordinates.

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x + 2y) \, dy \, dx$$

4. Use a double integral in polar coordinates to find the area of the region in the xy-plane inside the circle $x^2 + y^2 = 2$, above the line $y = 1$, and below the line $y\sqrt{3}x$.

5. Use a double integral to find the area of the 1st-quadrant region inside both circles $r = 3\sin\theta$ and $r = \sqrt{3}\cos\theta$.

6. Let S be the space region above the xy-plane and under the paraboloid $z = 16 - x^2 - y^2$. Set up the triple integral(s) necessary to compute the average value of $f(x, y, z) = 1 + x^2 + y^2 + z^3$ over S. Use technology to compute the average value.

7. Let T be the tetrahedron in space bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + 3z = 6$. Set up the triple integrals required to compute the average value of $f(x, y, z) = x + y + z$ on T. Use a computer algebra system to evaluate the integrals and state the average value.