Assignment 3 - Key

#1 $P(1,1,-1)$ $Q(2,0,2)$ $R(0,-2,1)$

$\vec{PQ} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{PR} = -\hat{i} - 3\hat{j} + 2\hat{k}$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix}$$

$$= \hat{i}(-2+9) - \hat{j}(2+3) + \hat{k}(-3-1)$$

$$= 7\hat{i} - 5\hat{j} - 4\hat{k}$$

Plane through P, Q, R:

$$7(x-2) - 5(y-0) - 4(z-2) = 0$$

or

$$\boxed{7x - 5y - 4z = 6}$$

#2 $P(1,-1,3)$, $\vec{n} = 3\hat{i} + \hat{j} + \hat{k}$

$$3(x-1) + (y+1) + (z-3) = 0$$

$$\boxed{3x + y + z = 5}$$

#3 $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z}{3} \Rightarrow$ $x = 2t+1$

$y = -t-1$

$z = 3t$

$3x + 2y - z = 5$

$\Rightarrow 3(2t+1) + 2(-t-1) - 3t = 5$

$t + 1 = 5 \Rightarrow t = 4$

POINT IS $x = 9$

$y = -5$ OR $\boxed{(9, -5, 12)}$

$z = 12$

#4 $\vec{n}_1 = -\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$\vec{n}_2 = 5\hat{\imath} - 2\hat{\jmath} - \hat{k}$

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\|\,\|\vec{n}_2\|} = \frac{3}{\sqrt{9}\sqrt{30}} = \frac{1}{\sqrt{30}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{30}}\right) \approx \boxed{79.48^\circ}$$

#5 $\quad -x - 2y + 2z = 5$

$5x - 2y - z = 0$

DIRECTION OF LINE OF INTERSECTION ?

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 2 \\ 5 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}(2+4) - \hat{j}(1-10) + \hat{k}(2+10)$$

$$= 6\hat{i} + 9\hat{j} + 12\hat{k}$$

A POINT ON LINE OF INTERSECTION ?

CHOOSE $x = 0$:

$$\begin{array}{r} -2y + 2z = 5 \\ -\ (-2y - z = 0) \\ \hline 3z = 5 \Rightarrow z = \frac{5}{3} \end{array}$$

$y = -\frac{5}{6}$

$x = 0$

$(0, -\frac{5}{6}, \frac{5}{3})$

$$\boxed{\begin{aligned} x &= 6t \\ y &= 9t - \tfrac{5}{6} \\ z &= 12t + \tfrac{5}{3} \end{aligned}}$$

#6 $P(1,2,3)$

$Q(3,2,1)$

$R(x,y,z)$

GIVEN PLANE HAS

$\vec{n} = 4\hat{i} - \hat{j} + 2\hat{k}$

$\vec{PR} = (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k}$

$\vec{QR} = (x-3)\hat{i} + (y-2)\hat{j} + (z-1)\hat{k}$

$$0 = \vec{n} \cdot (\vec{PR} \times \vec{QR}) = \begin{vmatrix} 4 & -1 & 2 \\ x-1 & y-2 & z-3 \\ x-3 & y-2 & z-1 \end{vmatrix}$$

$$= 4\left[(y-2)(z-1) - (y-2)(z-3)\right] + 1\left[(x-1)(z-1) - (x-3)(z-3)\right]$$

$$+ 2\left[(x-1)(y-2) - (x-3)(y-2)\right]$$

$$= 2x + 12y + 2z - 32$$

$$\boxed{x + 6y + z = 16}$$

#7 $P_1: 2x - 6y + 8z = 5$ $\quad \vec{n}_1 = 2\hat{i} - 6\hat{j} + 8\hat{k}$

$P_2: -x + 3y - 4z = 10$ $\quad \vec{n}_2 = -\hat{i} + 3\hat{j} - 4\hat{k}$

PLANES ARE PARALLEL

SINCE $\vec{n}_2 = -2\vec{n}_1$.

$(-3,1,-1)$ IS A POINT ON P_2.

DISTANCE FROM $(-3,1,-1)$ TO P_1 =

$$= \frac{|2(-3) - 6(1) + 8(-1) - 5|}{\sqrt{4 + 36 + 64}} = \boxed{\frac{25}{\sqrt{104}} \approx 2.45}$$

#8 $P(8,-3,2)$

$\ell: \frac{x-5}{2} = y-4 = \frac{z}{7}$

$Q(5,4,0)$ $\quad \vec{PQ} = -3\hat{i} + 7\hat{j} - 2\hat{k}$

$\vec{v} = 2\hat{i} + \hat{j} + 7\hat{k}$

$$\|\vec{PQ} \times \vec{v}\| = \left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 7 & -2 \\ 2 & 1 & 7 \end{vmatrix} \right\| = \sqrt{51^2 + 17^2 + 17^2} = \sqrt{3179}$$

$\|\vec{v}\| = \sqrt{54}$

$$\boxed{D = \sqrt{\frac{3179}{54}} \approx 7.67}$$

#9 $\vec{r}(t) = (3t+7)\hat{i} + 6t\hat{j} - (5-t)\hat{k}$

THE GRAPH OF $\vec{r}$ IS THE LINE PASSING THROUGH $(7, 0, -5)$ WITH DIRECTION VECTOR $\vec{v} = 3\hat{i} + 6\hat{j} + \hat{k}$.

$$\boxed{\hat{T}(t) = \frac{3\hat{i} + 6\hat{j} + \hat{k}}{\sqrt{46}}}$$

#10

$x = 2t^2$

$y = 1+3t \longrightarrow t = \frac{y-1}{3}$

$x = 2\left(\frac{y-1}{3}\right)^2$

$x = \frac{2}{9}(y-1)^2$

OR

$2(y-1)^2 - 9x = 0$