Math 233 - Assignment 4

Name _____

February 15, 2024

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 22.

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1. The velocity vector of a moving particle is given by

$$\vec{v}(t) = (\cos t)\,\hat{\imath} + (5\sin t)\,\hat{\jmath} + e^{-t}\,\hat{k}.$$

   Find the position vector $\vec{r}(t)$ if the particle's motion began (at $t = 0$) at the point $(2, 7, 4)$.

2. For $t \geq 0$, let $\vec{r}(t) = (\cos t + t\,\sin t)\,\hat{\imath} + (\sin t - t\,\cos t)\,\hat{\jmath}$. Compute the principal unit tangent vector, $\hat{T}(t)$.

3. Let $\vec{r}(t) = \cos 5t\,\hat{\imath} - t\,\hat{\jmath} - \sin 5t\,\hat{k}$. Starting from $t = 0$, find the arc-length parameter, $s(t)$, and then reparameterize $\vec{r}$ in terms of s. Then show that after the function is reparameterized, its derivative has magnitude 1.

4. Set up the definite integral that gives the length of the graph of

$$\vec{r}(t) = (3t^2 + 1)\,\hat{\imath} + (4t^2 - 1)\,\hat{\jmath} + 4t^3\,\hat{k}$$

   from $t = 0$ to $t = 2$. Evaluate your integral by hand. (If you've done everything correctly, your integral should require a simple u-substitution.)

5. Let $\vec{r}(t) = (t^2 - t)\,\hat{\imath} + \dfrac{1}{6}(4t - 1)^{3/2}\,\hat{\jmath} + 5\,\hat{k}$. Starting from $t = 1$, reparameterize $\vec{r}$ in terms of the arc-length parameter s.

6. Let $\vec{r}(t) = -\cos 3t\,\hat{\imath} - \sin 3t\,\hat{\jmath} + 4t\,\hat{k}$. Compute $\hat{N}(t)$.

7. For $-\pi/2 < x < \pi/2$, let $f(x) = \ln(\cos x)$. Compute the curvature function and say where the graph of f has its maximum curvature.

8. Find the curvature at the point P.

$$\vec{r}(t) = t\,\hat{\imath} + t^2\,\hat{\jmath} + \frac{t^3}{4}\,\hat{k}, \qquad P\,(2, 4, 2)$$

9. Let $\vec{r}(t) = t\,\hat{\imath} + \ln(\cos t)\,\hat{\jmath} + 5\,\hat{k}$. Compute $\hat{T}(t)$ and $\hat{N}(t)$.