## Math 233 - Assignment 5

February 22, 2024

Name $\qquad$
Score $\qquad$

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 29.

1. A golf ball is hit in a horizontal direction with speed $v_{0}$ off the top edge of a building that is 100 ft tall.
(a) Using $g=32 \mathrm{ft} / \mathrm{s}^{2}$ and ignoring air resistance, write the vector-valued function that gives the position of the golf ball at time $t$.
(b) Find $v_{0}$ if the ball lands 450 ft away from the building.
2. A projectile is launched from a bench above the ground. It is launched with an initial speed of 100 feet per second and at an angle of $30^{\circ}$ above the horizontal. The projectile hits the ground after it has covered a horizontal distance of 276 ft . How high is the bench?
3. Determine the domain of each function.
(a) $f(x, y)=4 \ln \left(y^{2}-x\right)$
(b) $z=\sqrt{100-4 x^{2}-25 y^{2}}$
4. Determine the range of $g(x, y)=\sqrt{16-4 x^{2}-y^{2}}$.
5. Describe the level surface $w(x, y, z)=4$ when $w(x, y, z)=x^{2}+y^{2}-z^{2}$.
6. In a field goal attempt on a flat field, a football is kicked off the ground at an angle of $30^{\circ}$ to the horizontal with an initial speed of $65 \mathrm{ft} / \mathrm{sec}$. (Ignore air resistance in this problem and use $g=32 \mathrm{ft} / \mathrm{s}^{2}$.)
(a) What horizontal distance does the football travel while it is in the air?
(b) Set up the definite integral that gives the length of the path of the football. Use your calculator to estimate the value of the integral.
(c) To score a field goal, the ball must clear the cross bar of the goal post, which is 10 ft above the ground. What is the furthest from the goal post the kick can originate and score a field goal?
7. Consider the surface described by the equation $4 x^{2}-y^{2}+9 z^{2}=4$.
(a) Fix a value for one of the variables and draw a good sketch of the corresponding level curve.
(b) Fix a value for one of the other variables and briefly describe the corresponding level curve.
(c) Identify the surface.
8. Consider the function $h(x, y)=\sqrt{1+x-y^{2}}$.
(a) What is the domain of $h$ ?
(b) Sketch the level curve $h(x, y)=0$.
(c) The graph of $h$ is one-half of one of the quadric surfaces that we are familiar with. Describe the graph of $h$.
9. Each of these equations defines a surface in 3-space. Describe each surface.
(a) $4 y=x^{2}+8 z^{2}$
(b) $z=9 x-7 y+13$
(c) $x^{2}+y^{2}=4$
(d) $2 x^{2}+8 y^{2}+z^{2}=16$
10. A particle moves in a path described by the $\vec{r}(t)=t^{2} \hat{\imath}+(2 t-3) \hat{\jmath}+\left(3 t^{2}-3 t\right) \hat{k}$, where $t$ is measured in seconds and distance is measured in feet. Find $a_{\hat{T}}$ and $a_{\hat{N}}$, the tangential and normal components of the acceleration.
