MTH 233 Assignment #5

1) a) $\boxed{\vec{r}(t) = V_0 t\ \hat{i} + (-16t^2 + 100)\hat{j}}$

b) $V_0 t = 450$ WHEN $\underbrace{-16t^2 + 100 = 0}$

$t = \sqrt{\frac{100}{16}} = 2.5$ SEC

$\boxed{V_0 = \frac{450}{2.5} = 180\ \text{FT/SEC}}$

2) $\vec{r}(t) = 100\cos 30^\circ\ t\ \hat{i} + (-16t^2 + 100\sin 30^\circ\ t + y_0)\hat{j}$

$\vec{r}(t) = 50\sqrt{3}\ t\ \hat{i} + (-16t^2 + 50t + y_0)\hat{j}$

$50\sqrt{3}\ t = 276$ WHEN $-16t^2 + 50t + y_0 = 0$

$t = \frac{276}{50\sqrt{3}} \approx 3.186973$ sec

$\boxed{y_0 = 16t^2 - 50t \approx 3.16\ \text{FT}}$

3 a) $f(x,y) = 4\ln(y^2 - x)$ MUST HAVE $y^2 - x > 0$ OR $x < y^2$

$\boxed{\text{DOMAIN} = \{(x,y) : x < y^2\}}$

b) $z = \sqrt{100 - 4x^2 - 25y^2}$ MUST HAVE $100 - 4x^2 - 25y^2 \geq 0$ OR $4x^2 + 25y^2 \leq 100$

$\boxed{\text{DOMAIN} = \{(x,y) : 4x^2 + 25y^2 \leq 100\}}$

4) $g(x,y) = \sqrt{16-4x^2-y^2}$ $\qquad \sqrt{0} \le g(x,y) \le \sqrt{16}$

$$\boxed{\text{RANGE} = [0,4]}$$

5) $x^2+y^2-z^2=4$

CROSS SECTIONS ARE HYPERBOLAS, HYPERBOLAS, CIRCLES AND CIRCULAR CROSS SECTIONS EXIST FOR ALL Z.

THE LEVEL SURFACE IS A CIRCULAR HYPERBOLOID OF ONE SHEET WITH CIRCULAR CROSS SECTIONS CENTERED ON THE Z-AXIS.

6) $\vec{r}(t) = 65\cos 30^\circ\, t\,\hat{i} + (-16t^2 + 65\sin 30^\circ\, t)\,\hat{j}$

$$\vec{r}(t) = \frac{65\sqrt{3}}{2}t\,\hat{i} + \left(-16t^2 + \frac{65}{2}t\right)\hat{j}$$

a) $-16t^2 + \frac{65}{2}t = 0 \Rightarrow -16t\left(t - \frac{65}{32}\right) = 0 \Rightarrow t = 0$ or $\underline{t = \frac{65}{32}}$

$$\frac{65\sqrt{3}}{2}\left(\frac{65}{32}\right) \approx \boxed{114.34 \text{ FT}}$$

b) $$\int_0^{65/32} \|\vec{r}\,'(t)\|\,dt = \int_0^{65/32} \sqrt{\left(\frac{65\sqrt{3}}{2}\right)^2 + \left(-32t + \frac{65}{2}\right)^2}\,dt$$

$$\approx \boxed{120.41 \text{ FT}}$$

PROBLEM 6 CONTINUED ...

c) MUST HAVE

$$-16t^2 + \frac{65}{2}t = 10 \Rightarrow t = \frac{-\frac{65}{2} \pm \sqrt{\left(\frac{65}{2}\right)^2 - 4(-16)(-10)}}{2(-16)}$$

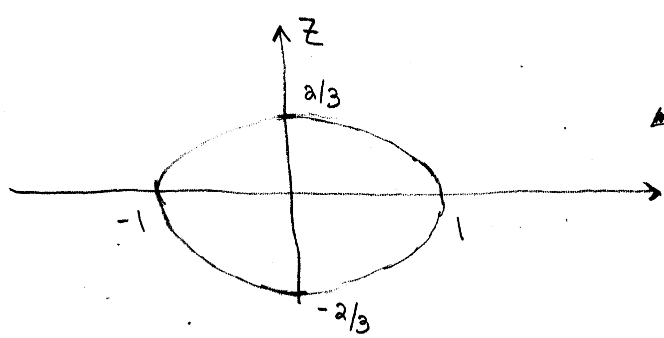
$$\approx 0.378056 \text{ OR } \underline{1.653194}$$

$$\frac{65\sqrt{3}}{2}(1.653194) \approx \boxed{93.06 \text{ FT}}$$

7) $4x^2 - y^2 + 9z^2 = 4$

a) $y = 0 \Rightarrow 4x^2 + 9z^2 = 4 \Rightarrow x^2 + \frac{z^2}{\left(\frac{2}{3}\right)^2} = 1$

ELLIPSE



b) $z = 0 \Rightarrow 4x^2 - y^2 = 4$

DESCRIBES A HYPERBOLA IN THE XY-PLANE.

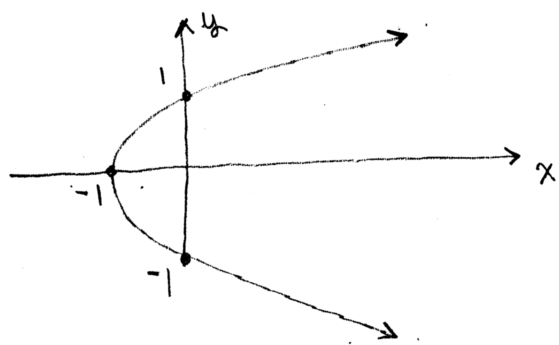
c) CROSS SECTIONS ARE HYPERBOLAS, HYPERBOLAS, ELLIPSES WITH ELLIPSES EXISTING FOR ANY y. $\Rightarrow$ (HYPERBOLOID ON ONE SHEET)

8) $h(x,y) = \sqrt{1 + x - y^2}$

a) MUST HAVE $1 + x - y^2 \geq 0$ OR $y^2 \leq x + 1$

$$\boxed{\text{DOMAIN} = \{ (x,y) : y^2 \leq x + 1 \}}$$

b) $h(x,y) = 0 \Rightarrow 1 + x - y^2 = 0 \Rightarrow x = y^2 - 1$



c) $z^2 = 1 + x - y^2$

OR $x = y^2 + z^2 - 1$

THE SURFACE IS THE UPPER HALF $(z > 0)$ OF THE CIRCULAR PARABOLOID OPENING UP THE POSITIVE X-AXIS WITH VERTEX AT $(-1, 0, 0)$.

9) a) $4y = x^2 + 8z^2$

CROSS SECTIONS ARE PARABOLAS, ELLIPSES, PARABOLAS.

THE SURFACE IS AN ELLIPTICAL PARABOLOID OPENING UP THE POSITIVE y-AXIS WITH VERTEX AT $(0,0,0)$.

b) $z = 9x - 7y + 13 \iff 9x - 7y - z = -13$

SURFACE IS THE PLANE WITH $\vec{n} = 9\hat{i} - 7\hat{j} - \hat{k}$ PASSING THROUGH $(0,0,13)$.

c) $x^2 + y^2 = 4$ $\quad z$ IS ARBITRARY. $z = 0$ CROSS SECTION IS A CIRCLE.

SURFACE IS A CIRCULAR CYLINDER CENTERED ON THE z-AXIS WITH RADIUS 2.

d) $2x^2 + 8y^2 + z^2 = 16$

SHOULD BE EASY TO RECOGNIZE

ELLIPSOID CENTERED AT $(0,0,0)$.

10) $\vec{r}(t) = t^2\,\hat{i} + (2t-3)\,\hat{j} + (3t^2-3t)\,\hat{k}$

$$\vec{r}\,'(t) = 2t\,\hat{i} + 2\hat{j} + (6t-3)\,\hat{k}$$

$$\vec{r}\,''(t) = 2\hat{i} + 6\hat{k}$$

$$a_T = \frac{\vec{r}\,'\cdot\vec{r}\,''}{\|\vec{r}\,'\|} = \frac{4t + 6(6t-3)}{\sqrt{4t^2+4+(6t-3)^2}} = \frac{40t-18}{\sqrt{40t^2-36t+13}}$$

$$\boxed{a_T = \frac{40t-18}{\sqrt{40t^2-36t+13}}}$$

$$\vec{r}\,'\times\vec{r}\,'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2t & 2 & 6t-3 \\ 2 & 0 & 6 \end{vmatrix} = \hat{i}\,(12) - \hat{j}\,(12t-12t+6) + \hat{k}(-4)$$

$$= 12\hat{i} - 6\hat{j} - 4\hat{k}$$

$$\|\vec{r}\,'\times\vec{r}\,''\| = \sqrt{144+36+16} = \sqrt{196} = 14$$

$$\boxed{a_N = \frac{14}{\sqrt{40t^2-36t+13}}}$$