Math 233 - Assignment 6

Name _____

February 29, 2024

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due March 7.

1. Use algebraic techniques to find the limit: $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

2. Show that the limit does not exist: $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4 + y^2}$

3. Show that the limit does not exist: $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy + y^3}{x^2 + y^2}$.

4. Show that the limit does not exist: $\lim\limits_{(x,y)\to(1,0)} \dfrac{(x-1)y + y^3}{(x-1)^2 + y^2}$.
   (Have you noticed that this is practically the same problem as #3?)

5. Find the limit or show that it does not exist.
   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 16y^2}{x^2 + 4y}$
   (b) $\lim\limits_{(x,y)\to(2,2)} \dfrac{3x - 3y}{\sqrt{x} - \sqrt{y}}$

6. Let $f(x, y) = e^{xy} + \sin(4x)\cos(5y)$. Find $f_x(x, y)$ and $f_y(x, y)$.

7. Let $z = \ln(xy + y^2)$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

8. Let $z = x^2 + 3xy + 2y^2$. Find $\dfrac{\partial^2 z}{\partial x^2}$ and $\dfrac{\partial^2 z}{\partial y^2}$.