# MTH 233 Assignment #6 key

1) $\lim_{(x,y)\to(0,0)} \dfrac{x(x-y)}{\sqrt{x}-\sqrt{y}} \cdot \dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

$$= \lim_{(x,y)\to(0,0)} \dfrac{x\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}{\cancel{(x-y)}}$$

$$= \lim_{(x,y)\to(0,0)} \left(x(\sqrt{x}+\sqrt{y})\right) = \boxed{0}$$

2) $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$

Along $x=0$: $\lim_{y\to 0} \dfrac{0}{y^2} = \lim_{y\to 0} 0 = 0$

Along $y=x^2$: $\lim_{x\to 0} \dfrac{x^4}{2x^4} = \lim_{x\to 0} \dfrac{1}{2} = \dfrac{1}{2}$

} Limit DNE by two-path test.

3) $\lim_{(x,y)\to(0,0)} \dfrac{xy+y^3}{x^2+y^2}$

Along $x=0$: $\lim_{y\to 0} \dfrac{y^3}{y^2} = \lim_{y\to 0} y = 0$

Along $x=y$: $\lim_{y\to 0} \dfrac{y^2+y^3}{2y^2} = \lim_{y\to 0} \dfrac{1+y}{2} = \dfrac{1}{2}$

} Limit DNE by Two Path Test

4) $\lim_{(x,y)\to(1,0)} \dfrac{(x-1)y+y^3}{(x-1)^2+y^2}$

Along $x=1$: $\lim_{y\to 0} \dfrac{y^3}{y^2} = \lim_{y\to 0} y = 0$

Along $y=x-1$: $\lim_{y\to 0} \dfrac{y^2+y^3}{2y^2} = \lim_{y\to 0} \dfrac{1+y}{2} = \dfrac{1}{2}$

} Limit DNE by Two Path Test.

★ Problems 3 & 4 are same under the substitution $u = x-1$

5)

a) $$\lim_{(x,y)\to(0,0)} \frac{x^4-16y^2}{x^2+4y} = \lim_{(x,y)\to(0,0)} \frac{(x^2+4y)(x^2-4y)}{x^2+4y}$$

$$= \lim_{(x,y)\to(0,0)} (x^2-4y) = \boxed{0}$$

b) $$\lim_{(x,y)\to(2,2)} \frac{3(x-y)}{\sqrt{x}-\sqrt{y}} \cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \lim_{(x,y)\to(2,2)} \frac{3(x-y)(\sqrt{x}+\sqrt{y})}{x-y}$$

$$= \lim_{(x,y)\to(2,2)} 3(\sqrt{x}+\sqrt{y}) = \boxed{6\sqrt{2}}$$

6) $f(x,y) = e^{xy} + \sin(4x)\cos(5y)$

$$f_x(x,y) = ye^{xy} + 4\cos(4x)\cos(5y)$$

$$f_y(x,y) = xe^{xy} - 5\sin(4x)\sin(5y)$$

7) $z = \ln(xy + y^2)$

$$\frac{\partial z}{\partial x} = \frac{y}{xy + y^2}$$

$$\frac{\partial z}{\partial y} = \frac{x + 2y}{xy + y^2}$$

8) $z = x^2 + 3xy + 2y^2$

$$\frac{\partial z}{\partial x} = 2x + 3y$$

$$\frac{\partial z}{\partial y} = 3x + 4y$$

$$\frac{\partial^2 z}{\partial x^2} = 2$$

$$\frac{\partial^2 z}{\partial y^2} = 4$$