

# Math 233 - Assignment 7

March 21, 2024

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. This assignment is due March 28.

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1. Use differentials to estimate the change in  $f(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}$  as  $(x, y, z)$  changes from  $(3, 4, 12)$  to  $(3.04, 4.08, 11.97)$ .
2. Use differentials to estimate the change in  $h(x, y, z) = \cos(\pi xy) + xz^2$  as  $(x, y, z)$  changes from  $(-1, -1, -1)$  to  $(-0.94, -0.95, -0.93)$ .
3. Use the definition of **differentiable** to show that  $f(x, y) = xy - xy^2$  is differentiable at any point in  $\mathbb{R}^2$ .
4. Find the linearization of  $f(x, y) = e^{2y-x}$  at  $(1, 2)$ . Then use your linearization to approximate  $f(0.95, 2.03)$ .
5. Use an appropriate linearization to estimate the value of  $(0.94)^2(4.03)^{1/2}(1.02)^5$ .
6. Use differentials to estimate the change in  $T = x(e^y + e^{-y})$  when  $x = 2$ ,  $y = \ln 2$ ,  $\Delta x = 0.1$ , and  $\Delta y = 0.02$ . (Notice that the estimated change in  $T$  is significantly bigger than the individual changes in  $x$  and  $y$ .)
7. Find an equation of the plane tangent to the surface  $z = 9 - x^2 - y^2$  at the point  $(1, 2, 4)$ .
8. Let  $f(x, y) = x^2 \sin(2y)$ . Find an equation of the plane tangent to the graph of  $f$  at the point  $(2, \pi/6)$ .
9. Suppose that  $w = 3xy + yz$  and that  $x$ ,  $y$ , and  $z$  are functions of  $u$  and  $v$  such that

$$x = \ln u + \cos v, \quad y = 1 + u \sin v, \quad z = uv.$$

Use the appropriate chain rule to find  $\partial w / \partial u$  at  $(u, v) = (1, \pi)$ .

10. Suppose  $w = f(x, y)$ , where  $x = u - v$  and  $y = v - u$ . Use the chain rule to show that  $\frac{\partial w}{\partial u} + \frac{\partial w}{\partial v} = 0$ .