MTH 233 - Assignment #7

(1) $w = f(x,y,z) = \ln\sqrt{x^2+y^2+z^2} = \frac{1}{2}\ln(x^2+y^2+z^2)$

$$f_x(x,y,z) = \frac{x}{x^2+y^2+z^2}, \quad f_y(x,y,z) = \frac{y}{x^2+y^2+z^2}, \quad f_z(x,y,z) = \frac{z}{x^2+y^2+z^2}$$

$$(x,y,z) = (3,4,12), \quad (\Delta x, \Delta y, \Delta z) = (0.04, 0.08, -0.03)$$

$$\Delta w \approx \frac{3}{169}(0.04) + \frac{4}{169}(0.08) + \frac{12}{169}(-0.03)$$

$$= \boxed{\frac{0.08}{169} \approx 0.000473}$$

(2) $h(x,y,z) = \cos(\pi xy) + xz^2$

$$h_x(x,y,z) = -\pi y\sin(\pi xy) + z^2, \quad h_y(x,y,z) = -\pi x\sin(\pi xy),$$

$$h_z(x,y,z) = 2xz$$

$$(x,y,z) = (-1,-1,-1), \quad (\Delta x, \Delta y, \Delta z) = (0.06, 0.05, 0.07)$$

$$\Delta w \approx 1(0.06) + 0(0.05) + 2(0.07) = \boxed{0.20}$$

3 $f(x,y) = xy - xy^2 \Rightarrow f_x(x,y) = y - y^2, \; f_y(x,y) = x - 2xy$

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y)$$

$$= (x+\Delta x)(y+\Delta y) - (x+\Delta x)(y+\Delta y)^2 - xy + xy^2$$

$$= \cancel{xy} + \underline{y\Delta x} + \underline{x\Delta y} + \underline{\underline{\Delta x \Delta y}} - \cancel{xy^2} - \underline{\underline{2xy\Delta y}} - \underline{\underline{x\Delta y^2}}$$
$$- \underline{y^2\Delta x} - \underline{\underline{2y\Delta x\Delta y}} - \underline{\underline{\Delta x \Delta y^2}} - \cancel{xy} + \cancel{xy^2}$$

$$= (y-y^2)\Delta x + (x-2xy)\Delta y + (\Delta y - 2y\Delta y - \Delta y^2)\Delta x - (x\Delta y)\Delta y$$

$$= f_x(x,y)\Delta x + f_y(x,y)\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

WHERE $\epsilon_1 = \Delta y - 2y\Delta y - \Delta y^2$

AND

$\epsilon_2 = -x\Delta y$

NOTICE THAT $\epsilon_1 \to 0$ AND $\epsilon_2 \to 0$ AS $(\Delta x, \Delta y) \to (0,0)$.

THE ABOVE HOLDS FOR ALL (x,y) IN $\mathbb{R}^2$.

$\Rightarrow$ f IS DIFFERENTIABLE AT ANY POINT IN $\mathbb{R}^2$.

4 $f(x,y) = e^{2y-x}$ $\quad f(1,2) = e^3$

$f_x(x,y) = -e^{2y-x}$, $\quad f_y(x,y) = 2e^{2y-x}$

$f_x(1,2) = -e^3$, $\quad f_y(1,2) = 2e^3$

$$\boxed{L(x,y) = e^3 - e^3(x-1) + 2e^3(y-2)}$$

$$f(0.95, 2.03) \approx e^3 + 0.05e^3 + 0.06e^3 = \boxed{1.11e^3 \approx 22.295}$$

5 Let $f(x,y,z) = x^2 y^{1/2} z^5$

$f_x(x,y,z) = 2xy^{1/2}z^5$, $\quad f_y(x,y,z) = \dfrac{x^2 z^5}{2\sqrt{y}}$, $\quad f_z(x,y,z) = 5x^2y^{1/2}z^4$

$(x_0, y_0, z_0) = (1,4,1)$

$$L(x,y,z) = f(1,4,1) + f_x(1,4,1)(x-1) + f_y(1,4,1)(y-4) + f_z(1,4,1)(z-1)$$

$$= 2 + 4(x-1) + \frac{1}{4}(y-4) + 10(z-1)$$

$$f(0.94, 4.03, 1.02) \approx 2 + 4(-0.06) + \frac{1}{4}(0.03) + 10(0.02)$$

$$= 2 - 0.24 + 0.0075 + 0.2$$

$$= \boxed{1.9675}$$

6. $T = x(e^y + e^{-y})$

$T_x = e^y + e^{-y}$, $T_y = x(e^y - e^{-y})$

$\Delta T \approx (e^y + e^{-y})\Delta x + x(e^y - e^{-y})\Delta y$

$x = 2$, $y = \ln 2$, $\Delta x = 0.1$, $\Delta y = 0.02$

$\Delta T \approx \left(2 + \frac{1}{2}\right)(0.1) + 2\left(2 - \frac{1}{2}\right)(0.02)$

$= 0.25 + 0.06 = 0.31$

$\boxed{\Delta T \approx 0.31}$

7. $z = f(x,y) = 9 - x^2 - y^2$ $\quad f(1,2) = 4$

$f_x(x,y) = -2x$, $f_y(x,y) = -2y$

$f_x(1,2) = -2$; $f_y(1,2) = -4$

$L(x,y) = 4 - 2(x-1) - 4(y-2)$

TANGENT PLANE IS

$$z = 4 - 2(x-1) - 4(y-2)$$

OR

$$2x + 4y + z = 14$$

8.

$z = f(x,y) = x^2 \sin 2y$ $\quad f(2, \pi/6) = 4 \sin \frac{\pi}{3} = 2\sqrt{3}$

$f_x(x,y) = 2x \sin 2y$, $f_y(x,y) = 2x^2 \cos 2y$

$f_x(2, \pi/6) = 4 \sin \frac{\pi}{3} = 2\sqrt{3}$, $f_y(2, \pi/6) = 8 \cos \frac{\pi}{3} = 4$

$L(x,y) = 2\sqrt{3} + 2\sqrt{3}(x-2) + 4\left(y - \frac{\pi}{6}\right)$

TANGENT PLANE IS

$$z = 2\sqrt{3} + 2\sqrt{3}(x-2) + 4\left(y - \frac{\pi}{6}\right)$$

9. $w = 3xy + yz$

$x = \ln u + \cos v$

$y = 1 + u \sin v$

$z = uv$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$= (3y)\left(\frac{1}{u}\right) + (3x+z)(\sin v) + (y)(v)$$

WHEN $(u,v) = (1,\pi)$, $x = -1$, $y = 1$, AND $z = \pi$,

SO $$\frac{\partial w}{\partial u} = (3)(1) + (-3+\pi)(0) + (1)(\pi)$$

$$= \boxed{3+\pi}$$

10.

$$w = f(x,y)$$

$$x = u - v$$

$$y = v - u$$

ADD

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} = \frac{\partial w}{\partial x}(1) + \frac{\partial w}{\partial y}(-1)$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} = \frac{\partial w}{\partial x}(-1) + \frac{\partial w}{\partial y}(1)$$

ADD

$$\frac{\partial w}{\partial u} + \frac{\partial w}{\partial v} = 0 \quad \checkmark$$