Math 233 - Assignment 8

Name _____

March 28, 2024

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 4.

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1. Find dy/dx if $y^2 - x^2 - \sin xy = 0$.

2. Find $\partial z/\partial x$ and $\partial z/\partial y$ at $(0, 0, 0)$ if $x^3 + z^2 + ye^{xz} + z\cos y = 0$.

3. Suppose we would like to convert the differentiable function $w = f(x, y)$ to polar coordinates by using $x = r\cos\theta$ and $y = r\sin\theta$.

   (a) Show that
   $$\frac{\partial w}{\partial r} = f_x \cos\theta + f_y \sin\theta$$
   and
   $$\frac{1}{r}\frac{\partial w}{\partial \theta} = -f_x \sin\theta + f_y \cos\theta.$$

   (b) Solve the equations in part (a) for f_x and f_y in terms of $\partial w/\partial r$ and $\partial w/\partial\theta$.

   (c) Show that
   $$(f_x)^2 + (f_y)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2.$$
   (This expression is called the *Laplacian* of f.)

4. Find the directional derivative of $g(x, y) = \dfrac{x - y}{xy + 2}$ at the point $(1, -1)$ in the direction of $\vec{v} = 12\hat{\imath} + 5\hat{\jmath}$.

5. Find the gradient vector at $(-1, 2, -2)$: $\quad f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2} + \ln(xyz)$.

6. Find the gradient vector at the given point. Then sketch the gradient together with the level curve that passes through the point.
   $$f(x, y) = \tan^{-1}\frac{\sqrt{x}}{y}, \qquad (4, -2)$$

7. The electric voltage in a certain region in space is described by the function $V(x, y, z) = 5x^2 - 3xy + xyz$. At the point $(3, 4, 5)$, in what direction is the voltage increasing most rapidly? Give your answer as a unit vector.

8. Find a set of parametric equations for the line normal to the graph of
   $$x^2 - 8xyz + y^2 + 6z^2 = 0$$
   at the point $P\,(1, 1, 1)$.

9. Find an equation of the plane tangent to the surface $\sin(xz) = 4\cos(yz)$ at the point $(\pi, \pi/2, 1)$.

10. Let $G(x, y, z) = \dfrac{x}{z} + \dfrac{z}{y^2}$. Find a unit vector in the direction in which G decreases most rapidly at $P\,(1, 2, -2)$. What is the corresponding rate of decrease?