Math 233 - Assignment 9

April 4, 2024

Name ____________________

Score ____________________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 11.

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1. Find the critical points of $f(x,y) = 15x^3 - 3xy + 15y^3$.

2. Find and classify the critical points of $f(x,y) = 9 - x^4y^4$.

3. Find and classify the critical points of $g(x,y) = x^2 + x - 3xy + y^3 - 5$.

4. Find and classify the critical points of $f(x,y) = x^3 - 2xy + xy^2 - 7$.

5. It is easy to see that any point of the form $(0,b)$ or $(a,0)$ is a critical point of $f(x,y) = x^2y^2$. However, the second derivative test is inconclusive at each of these points since they give rise to zero determinants. Nonetheless, each point yields a minimum value. How can we be so sure of this?

6. In this problem you will find the extreme values of $f(x,y,z) = x^2 - y + yz$ subject to the constraint $x + y = z^2$.

   (a) Set up, but do not solve, the system of equations that is obtained by applying the Lagrange multiplier method to this problem.

   (b) Your system of equations has two solutions for (x,y,z). They are

   $$\left(-\frac{1}{3}, \frac{4}{9}, \frac{1}{3}\right) \qquad \text{and} \qquad \left(-\frac{1}{4}, \frac{1}{2}, \frac{1}{2}\right)$$

   Use this information to find the maximum and minimum values of $f(x,y,z)$ on the constraint surface.

   (c) Find the value of the Lagrange multiplier associated with each critical point.

7. Use Lagrange multipliers to find the extreme values of $f(x,y) = y^2 - 4x$ subject to $x^2 + y^2 = 9$.

8. Use Lagrange multipliers to find the extreme values of $f(x,y) = x^2y$ subject to the constraint $x^2 + y^2 = 1$.