Math 233 - Test 1

February 8, 2024

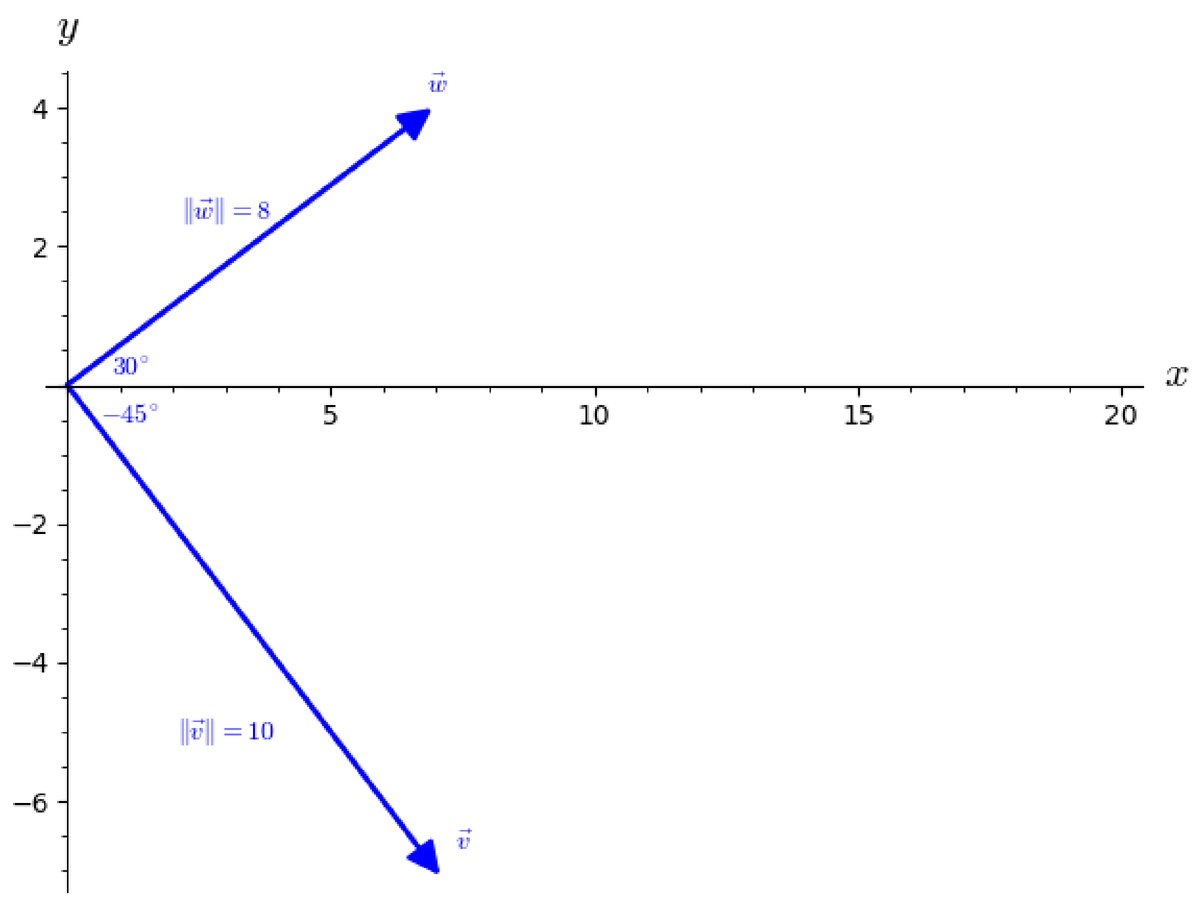
Name ____________________

Score __________

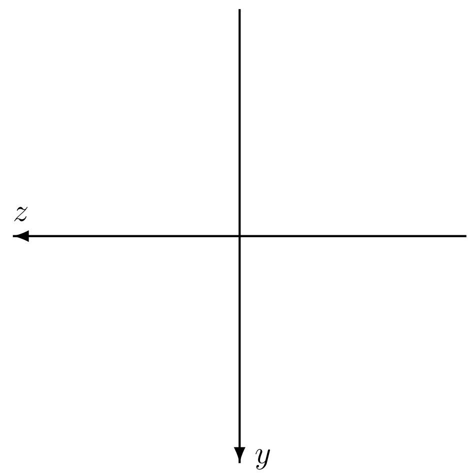
Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Determine a vector in 2-dimensional space that is perpendicular to $\vec{u} = 3\hat{\imath} - 5\hat{\jmath}$ and has magnitude 4.

2. (8 points) Referring to the figure below, sketch $\vec{w} + \vec{v}$. Then find the component form of $\vec{w} + \vec{v}$.



3. (3 points) A 3-dimensional rectangular coordinate system is set up in such a way that the positive y and z axes are as indicated below. Describe the direction of the positive x axis, and say how you know.



4. (10 points) Consider the line segment from $P\,(3, -2, 1)$ to $Q\,(6, -6, -8)$.

   (a) Find the midpoint of the line segment.

   (b) Find the length of the segment.

   (c) Find a set of parametric equations for the line **segment**.

   (d) Find symmetric equations for the line through P and Q.

5. (8 points) Let $\vec{x} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{y} = 3\hat{\imath} + 8\hat{\jmath} - 4\hat{k}$.

   (a) Let $\vec{w}$ be the projection of $\vec{y}$ onto $\vec{x}$. Compute $\vec{w}$.

   (b) Let $\vec{z} = \vec{y} - \vec{w}$ and compute $\vec{z}$.

   (c) Now compute $\vec{w} \cdot \vec{z}$. What does your answer say about $\vec{w}$ and $\vec{z}$?

6. (8 points) Find a vector of magnitude 10 that is orthogonal to both $\vec{a} = \langle 2, 4, -1, \rangle$ and $\vec{b} = \langle 1, 2, 1 \rangle$.

7. (6 points) Let $\vec{w} = \frac{2}{11}\hat{\imath} + \frac{6}{11}\hat{\jmath} + \frac{9}{11}\hat{k}$.

   (a) Confirm that $\vec{w}$ is a unit vector.

   (b) Show that each component of $\vec{w}$ is the cosine of the angle between $\vec{w}$ and the corresponding coordinate axis.

8. (16 points) Consider the points $A\,(3,5,-7)$, $B\,(6,-1,3)$, and $C\,(-2,-2,8)$.

   (a) Show that the points are NOT collinear.

   (b) Find the area of $\triangle ABC$.

   (c) Find an equation of the plane containing the points A, B, and C.

   (d) Find the measure of the angle between your plane in part (c) and the plane $2x - 7y + 6z = 0$. (If you were unable to do part (c), just thoroughly explain how you would find this angle.)

9. (5 points) Find a vector-valued function whose graph is the line described by

$$x = 2y - 6 = \frac{7 - z}{3}.$$

10. (8 points) The distance from a line to a point not on the line is given by $\dfrac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|}$, where $\vec{v}$ is a vector parallel to the line, P is a point on the line, and Q is the point not on the line.

Find the distance from the line

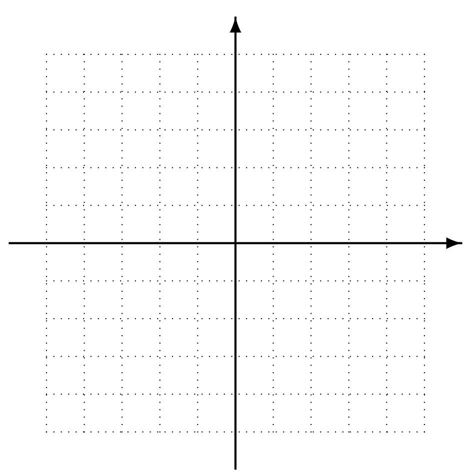
$$x = 2t + 7, \qquad y = -t - 3, \qquad z = 9$$

to the point $(5, -4, 3)$.

11. (8 points) Let $\vec{r}(t) = (\frac{1}{2}t + 1)\,\hat{\imath} + (3 - \frac{1}{4}t^2)\,\hat{\jmath}$.

(a) Write the set of parametric equations whose graph is that of $\vec{r}(t)$ Then eliminate the parameter t to obtain an equation in the rectangular coordinates x and y.

(b) Carefully sketch the graph of $\vec{r}(t)$ and draw arrows to show the graph's orientation.



12. (8 points) Let $\vec{r}(t) = \sin 2t\,\hat{\imath} + \cos 2t\,\hat{\jmath} + t\,\hat{k}$.

(a) Describe the graph of $\vec{r}(t)$.

(b) Compute $\|\vec{r}(t)\|$.

(c) Find $\vec{r}\,'(t)$.

(d) Show that $\vec{r}(t)$ is orthogonal to $\vec{r}\,'(t)$ only when $t = 0$.

13. (6 points) Let $\vec{r}(t) = \dfrac{t-2}{t^2-4}\,\hat{\imath} + \dfrac{\sin t}{t}\,\hat{\jmath} + |t|\,\hat{k}$.

(a) For which values of t is $\vec{r}(t)$ discontinuous?

(b) Which of those discontinuities are removable? Briefly say how you know.