Math 233 - Test 2

March 7, 2024

Name ______________________

Score ____________

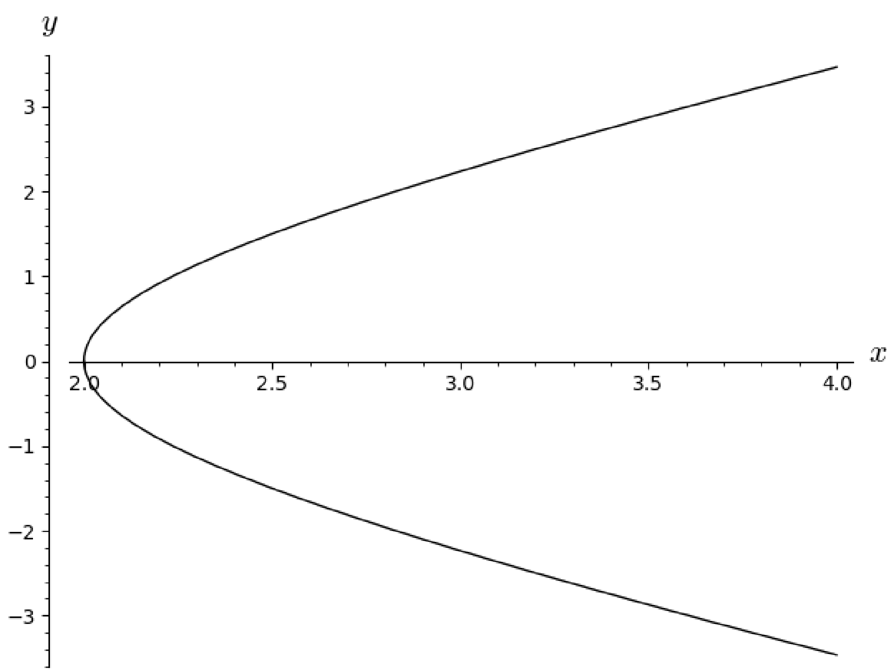
Show all work to receive full credit. Supply explanations where necessary.

1. (9 points) Let $\vec{r}(t) = \left(\frac{1}{t} + t\right)\hat{\imath} + \left(\frac{1}{t} - t\right)\hat{\jmath}$.

   (a) Show that $\|\vec{r}\,'(t)\| = \dfrac{\sqrt{2t^4+2}}{t^2}$.

   (b) Find $\hat{T}(2)$.

   (c) The graph of r is shown below. Sketch $\hat{T}(2)$ at the point where $t = 2$.



2. (6 points) Find $\vec{r}(t)$ given the following information.

$$\vec{r}\,'(t) = \frac{3}{2}(t+1)^{1/2}\,\hat{\imath} + e^{-t}\,\hat{\jmath} + \frac{1}{t+1}\,\hat{k}, \quad \vec{r}(0) = \hat{k}$$

3. (8 points) Find the length of the graph of

$$\vec{r}(t) = (1-t^2)\,\hat{\imath} + \sqrt{8}\,t^2\,\hat{\jmath} + (7+2t^3)\,\hat{k}$$

from $t = 0$ to $t = 1$. (Do not use your calculator.)

4. (8 points) Let $\vec{r}(t) = (6\sin 2t)\,\hat{\imath} + 5t\,\hat{\jmath} + (6\cos 2t)\hat{k}$. Starting from $t = 0$, find the arc-length parameter $s(t)$, and then reparameterize $\vec{r}$ in terms of s.

5. (12 points) A volleyball is hit when it is 4 ft above the ground and 12 ft from a 6-ft-high net. It leaves the point of impact with an initial speed of 35 ft/sec at an angle of 27°.

(a) Find the vector-valued function that gives the position of the volleyball at time t. (Use $g = 32\ \text{ft/sec}^2$.)

(b) When does the volleyball reach its maximum height, and what is the maximum height?

(c) How high is the volleyball when it passes over the net?

(d) Find the range and the flight time of the volleyball.

6. (8 points) Compute the curvature of the graph of $\vec{r}(t) = \dfrac{t^3}{3}\,\hat{\imath} + \dfrac{t^2}{2}\,\hat{\jmath} + t\,\hat{k}$ at the point $(1/3, 1/2, 1)$.

7. (4 points) Look back at the graph of $\vec{r}$ in problem #1. By using the graph of $\vec{r}$, determine if there are any points at which $\hat{N}(t)$ is not defined. Explain.

8. (2 points) Suppose r is a curve for which $\hat{N}(t)$ exists for every t. Is it possible that $\hat{N}(t_0) = \vec{0}$ for some number t_0? Explain.

9. (3 points) What are the three important characteristics of the principal unit tangent vector?

10. (12 points) Each of these equations defines a surface in 3-space. Briefly describe each surface. If possible, do more than just give the name of each surface.

    (a) $3x^2 - 9y^2 - 8z^2 = -5$

    (b) $4x^2 + 4y^2 = 1$

    (c) $-x^2 - 2y^2 + 3z^2 = 4$

    (d) $(x-2)^2 + (y+3)^2 + z^2 = 16$

11. (4 points) Find and sketch the domain of the function $g(x, y) = \dfrac{\sin(xy)}{\sqrt{y - x^2 + 1}}$.

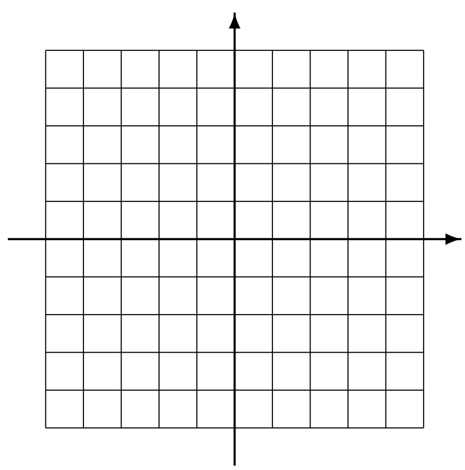
12. (6 points) Let $f(x, y, z) = \ln(x^2 + y^2 + z^2)$.

(a) What is the domain of f?

(b) What is the range of f?

(c) Describe the level surface $f(x, y, z) = 5$.

13. (3 points) Let $f(x, y) = x + y - 1$. Sketch the level curves $f(x, y) = c$ at levels $c = 0$, $c = 1$, and $c = 2$. Label the levels.



14. (15 points) Find each limit or show that it does not exist.

(a) $$\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2}$$

(b) $$\lim_{(x,y)\to(1,-1)} \frac{xy+1}{x^2-y^2}$$

(c) $$\lim_{(x,y)\to(3,3)} \frac{x-y}{x^4-y^4}$$