Math 233 - Test 3

April 11, 2024

Name ______________________

Score ____________

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) Find f_x, f_y, and f_z when $f(x, y, z) = yz\ln(xy^2)$.

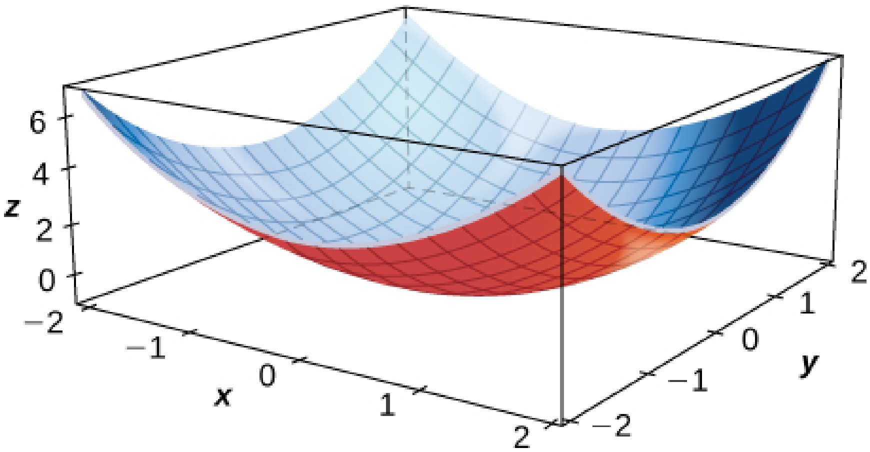
2. (6 points) Let $f(x, y) = x\cos(2y) + e^{y^3}$.

   (a) Which partial mixed partial derivative is less work to compute, f_{xy} or f_{yx}? Why?

   (b) Compute whichever mixed partial you said in part (a).

   (c) In this problem, would you expect $f_{xy} = f_{yx}$? Why or why not?

3. (4 points) Look carefully at the graph of $z = f(x, y)$ shown below. Use the graph to determine the sign of $f_x(1, -1)$ and $f_y(1, -1)$. Briefly explain.



4. (10 points) Find the linearization of $f(x, y, z) = \dfrac{\sin(xy)}{z}$ at the point $(x, y, z) = (\sqrt{\pi}/2, \sqrt{\pi}/3, \sqrt{\pi})$. Then use your linearization to approximate $f(0.9, 0.6, 1.7)$.

5. (6 points) Use differentials to approximate the change in $g(x, y) = \tan^{-1}(xy^2)$ as (x, y) changes from $(1, 1)$ to $(1.02, 0.97)$.

6. (4 points) Let $z = x^2(y+1)$. Compute the total differential dz. Near the point $(1, 0)$, is z more sensitive to changes in x or to changes in y? Explain.

7. (4 points) Suppose $v = f(w, x, y, z)$ is differentiable, and w, x, y, and z are differentiable functions of s and t. Write the formula for the derivative of v with respect to t.

8. (4 points) Use partial derivatives to find $\partial z/\partial y$ at the point $(1, 1, 1)$.

$$z^3 - xy + yz + y^3 - 2 = 0$$

9. (4 points) The voltage V in a circuit satisfies $V = IR$, where I is the current and R is the resistance. The voltage is dropping as the battery wears out and the resistor heats up. Find a formula for dV/dt, and use it to determine dI/dt when $R = 600\,\text{ohms}$, $I = 0.04\,\text{amp}$, $dR/dt = 0.5\,\text{ohm/sec}$, and $dV/dt = -0.01\,\text{volt/sec}$.

10. (8 points) Find a set of parametric equations for the line normal to the surface $z = 2e^{4x^2+2xy-4y}$ at the point $(1, 2, 2)$.

11. (6 points) Find the directional derivative of $g(x, y) = \dfrac{x - y}{xy + 2}$ at the point $(1, -1)$ in the direction of $\vec{v} = 12\hat{\imath} + 5\hat{\jmath}$.

12. (8 points) Let $G(x, y, z) = \dfrac{x}{z} + \dfrac{z}{y^2}$. Find a unit vector in the direction in which G decreases most rapidly at $P\,(1, 2, -2)$. What is the corresponding rate of decrease?

13. (12 points) Find the critical points of $f(x, y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$. Then use the 2nd partials test to classify your critical points.

14. (4 points) Suppose you are using the 2nd partials test and it is inconclusive. Describe two things that you might do in order to help you draw a conclusion.

15. (12 points) Use Lagrange multipliers to find the minimum and maximum values of $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

16. (4 points) Refer back to the problem above. Explain how we can be certain that the minimum and maximum values exist even before starting work on the problem.