Math 233 - Final Exam A

May 3, 2024

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due May 9. You must work individually.

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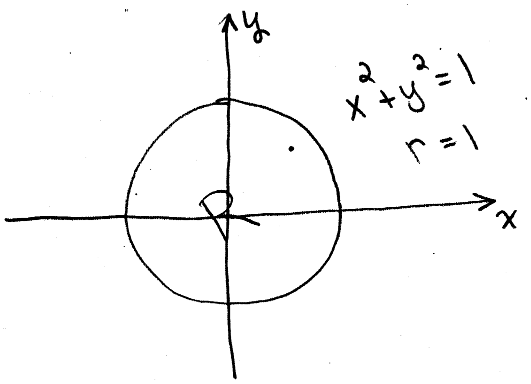
1. (10 points) Let E be the space region inside the cylinder $x^2 + y^2 = 1$ and between the two parallel planes $x + y + z = 1$ and $x + y + z = 3$. Find

$$\iiint_E xy\, dV$$

by setting up and evaluating an iterated integral in cylindrical coordinates. Show all work.

Z-AXIS COMES OUT OF PAPER



LOOKING DOWN THE Z-AXIS, THE PLANE $X+Y+Z=1$ IS 2 UNITS BELOW $X+Y+Z=3$.

$$\iiint_E xy\, dV = \iint_R \int_{z=1-x-y}^{z=3-x-y} xy\, dz\, dA$$

$$= \int_0^{2\pi}\int_0^1\int_{1-\cos\theta-\sin\theta}^{3-\cos\theta-\sin\theta} r^3\cos\theta\sin\theta\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 2r^3\cos\theta\sin\theta\, dr\, d\theta = \frac{1}{2}\int_0^{2\pi}\cos\theta\sin\theta\, d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

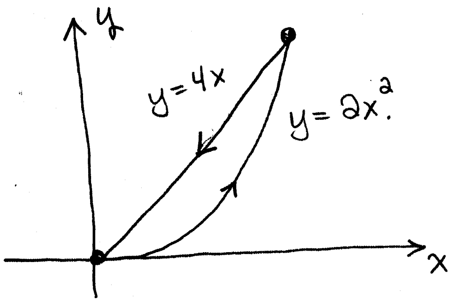
$$= \frac{1}{2}\int_0^0 u\, du = \boxed{0}$$

2. (10 points) Let C be the positively-oriented boundary of the plane region enclosed by the graphs of $y = 4x$ and $y = 2x^2$. Use Green's theorem to find

$$\int_C (y^2 - 2xy)\,dx + x^2\,dy.$$

Show all work.

$$= \iint_R (2x - 2y + 2x)\,dA$$



$$= \int_{x=0}^{x=2} \int_{y=2x^2}^{y=4x} (4x - 2y)\,dy\,dx$$

$2x^2 = 4x$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0,\ x = 2$

$$= \int_0^2 4xy - y^2 \Big|_{y=2x^2}^{y=4x} dx$$

$$= \int_0^2 (16x^2 - 16x^2 - 8x^3 + 4x^4)\,dx$$

$$= -2x^4 + \frac{4}{5}x^5 \Big|_0^2 = -32 + \frac{128}{5}$$

$$= \boxed{-\frac{32}{5}}$$

3. (10 points) Find the critical points of f. Then use the 2nd partials test to classify those critical points and determine all relative extrema and saddle points. Show all work.

$$f(x,y) = x^3 + y^2 + 2xy - 4x - 3y + 5$$

$f_x(x,y) = 3x^2 + 2y - 4 = 0 \rightarrow 2y = 4 - 3x^2$

$f_y(x,y) = 2y + 2x - 3 = 0$

$4 - 3x^2 + 2x - 3 = 0$

$\Downarrow$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3}, \quad x = 1$

$2y = 4 - \frac{1}{3}$, $y = \frac{11}{6}$ $\qquad$ $2y = 1$, $y = \frac{1}{2}$

$\left(-\frac{1}{3}, \frac{11}{6}\right)$ $\quad$ $\left(1, \frac{1}{2}\right)$

Two CRIT PTS

$$D(x,y) = \begin{vmatrix} 6x & 2 \\ 2 & 2 \end{vmatrix} = 12x - 4$$

$\left(-\frac{1}{3}, \frac{11}{6}\right) \ldots$

$D = -8 \Rightarrow \left(-\frac{1}{3}, \frac{11}{6}, \frac{317}{108}\right)$ IS A SADDLE PT.

$\left(1, \frac{1}{2}\right) \ldots$

$D = 8, \quad f_{xx}\left(1, \frac{1}{2}\right) = 6$

$f\left(1, \frac{1}{2}\right) = \frac{7}{4}$ IS A REL MINIMUM.