# Math 233 - Final Exam B

Name ____________________________

May 9, 2024

Score ___________

**Show all work to receive full credit**. Supply explanations where necessary.

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1. (10 points) A golf ball is hit from the ground toward a vertical cliff that is 150 m away. The ball is launched at a $40^\circ$ angle with respect to the horizontal, and its initial speed is 70 m/s. At what height will the ball strike the cliff? Will the ball ever reach its maximum possible height? Explain. (Use $g = 9.81\,\mathrm{m/s^2}$.)

2. (10 points) Find each limit or show that it does not exist.

(a) $$\lim_{(x,y)\to(2,1)} \frac{x-y-1}{\sqrt{x-y}-1}$$

(b) $$\lim_{(x,y)\to(2,1)} \frac{(x-2)(y-1)}{(x-2)^2+(y-1)^2}$$

3. (10 points) Let $w = xyz$.

(a) Compute the total differential $dw$.

(b) Use differentials to estimate the change in $w$ as $(x, y, z)$ changes from $(5, 3, 2)$ to $(5.1, 3.1, 2.1)$.

(c) Your answer is part (b) is an approximation for the volume of the walls of a empty box with inside dimensions 5 m by 3 m by 2 m, when the walls are 5 cm thick. Explain or illustrate this idea.

4. (10 points) Consider the surface described by the equation $z = 2e^{4x^2+2xy-4y}$.

   (a) Find an equation of the plane tangent to the surface at the point $(1, 2, 2)$.

   (b) Find a set of parametric equations for the line normal to the surface at the point $(1, 2, 2)$.

5. (10 points) Consider the double integral $\displaystyle\iint_R \frac{\sin x}{x}\, dA$, where $R$ is the triangular region in the $xy$-plane bounded by the $x$-axis, the line $y = x$, and the line $x = 1$. Sketch the region $R$, and set up the corresponding iterated integrals with both orders of integration. Then choose one of your iterated integrals and evaluate it.

6. (10 points) A region in space lies in the first octant (where $x, y, z \geq 0$) where it is bounded by the cylinder $y = x - x^2$ and the planes $z = 0$ and $z = y$. The volume of the region is $1/60\,\text{units}^3$. Use a triple integral to find the average value of $f(x, y, z) = 2x^2$ over the region.

7. (10 points) Let $C$ be the curve made up of two line segments: the first from $(1,3)$ to $(3,7)$, and the second from $(3,7)$ to $(3,10)$. Evaluate $\displaystyle\int_C \vec{F}(x,y)\cdot d\vec{r}$, where $\vec{F}(x,y) = (x^2y+2)\hat{\imath} + (1-xy)\hat{\jmath}$.