Math 233 - Quiz 10

April 23, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Find and classify the critical points of $g(x,y) = x^2 + x - 3xy + y^3 - 5$. Identify all relative extreme values.

$g_x(x,y) = 2x + 1 - 3y = 0$

$g_y(x,y) = -3x + 3y^2 = 0$

$x = y^2$

$2y^2 - 3y + 1 = 0$

$(2y-1)(y-1) = 0$

$y = \frac{1}{2}, \; y = 1$

$x = \frac{1}{4} \qquad x = 1$

$D(x,y) = \begin{vmatrix} 2 & -3 \\ -3 & 6y \end{vmatrix} = 12y - 9$

$\left(\frac{1}{4}, \frac{1}{2}\right)$: $D\left(\frac{1}{4}, \frac{1}{2}\right) = -3$

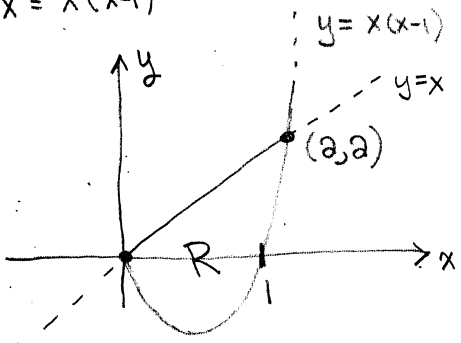
$\left(\frac{1}{4}, \frac{1}{2}, -\frac{79}{16}\right)$ IS A SADDLE POINT.

$(1,1)$: $D(1,1) = 3 > 0 \quad g_{xx}(1,1) = 2 > 0$

$g(1,1) = -5$ IS A REL. MIN

2. (5 points) Sketch the region of integration and evaluate the iterated integral.

$y = x^2 - x = x(x-1)$



$x^2 - x = x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, \; x = 2$

$\int_0^2 \int_{x^2-x}^{x} x \, dy \, dx$

$= \int_0^2 xy \Big|_{y=x^2-x}^{y=x} dx$

$= \int_0^2 \left(x^2 - x^3 + x^2\right) dx = \int_0^2 \left(2x^2 - x^3\right) dx$

$= \frac{2}{3}x^3 - \frac{1}{4}x^4 \Big|_{x=0}^{x=2}$

$= \frac{16}{3} - \frac{16}{4} = \frac{16}{12} = \boxed{\frac{4}{3}}$