Math 233 - Quiz 11

April 30, 2026

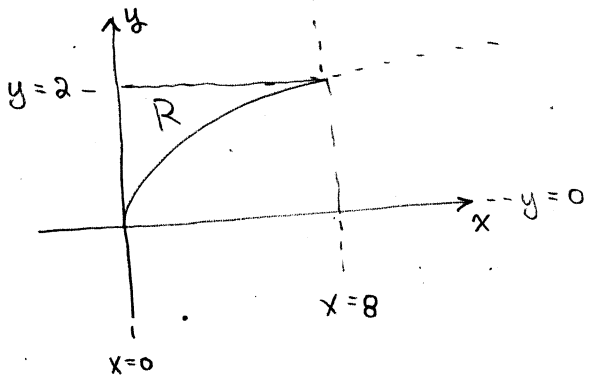
Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (5 points) Sketch the region of integration, reverse the order of integration, and evaluate the iterated integral.

$$\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\, dy\, dx$$

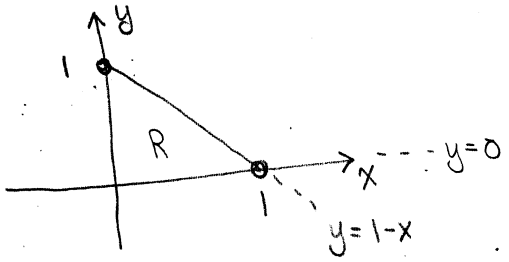


$$\int_{y=0}^{y=2} \int_{x=0}^{x=y^3} \frac{1}{y^4+1}\, dx\, dy$$

$$= \int_0^2 \frac{y^3}{y^4+1}\, dy \qquad u = y^4+1,\ du = 4y^3\, dy$$

$$= \frac{1}{4}\int_1^{17} \frac{1}{u}\, du = \boxed{\frac{1}{4}\ln 17}$$

2. (5 points) R is the region in the first quadrant that lies under the graph of $x + y = 1$. Find the average value of $f(x, y) = xy$ over R.



$$\iint_R xy\, dA = \int_{x=0}^{x=1} \int_{y=0}^{y=1-x} xy\, dy\, dx = \int_0^1 \frac{1}{2}x(1-x)^2\, dx$$

$$= \int_0^1 \left(\frac{1}{2}x - x^2 + \frac{1}{2}x^3\right) = \frac{1}{4} - \frac{1}{3} + \frac{1}{8} = \frac{1}{24}$$

Area of R $= \frac{1}{2}(1)(1) = \frac{1}{2}$

Average value $= \dfrac{1/24}{1/2} = \boxed{\frac{1}{12}}$