Math 233 - Quiz 12

May 7, 2026

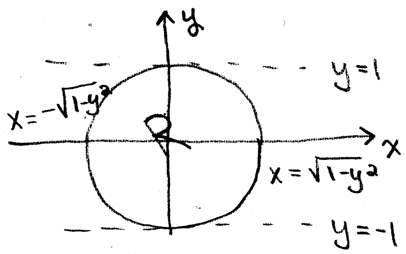
Name key

Score ____________

Show all work to receive full credit. Supply explanations when necessary. This quiz is due Tuesday, May 12. Use extra paper as necessary. Evaluate all integrals by hand.

1. (3 points) Sketch the region of integration, rewrite as an iterated integral in polar coordinates, and evaluate the polar integral.

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)\,dx\,dy\,.$$



UNIT CIRCLE

$x^2+y^2=1$

$r^2=1$

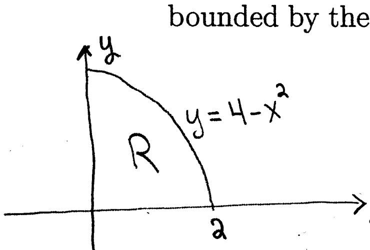
$$\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=1} r\ln(r^2+1)\,dr\,d\theta = 2\pi\int_0^1 r\ln(r^2+1)\,dr$$

$u=r^2+1$

$du=2r\,dr$

$$=\pi\int_1^2 \ln u\,du \xrightarrow{\text{OVER}}$$

2. (3 points) Use a triple integral to find the volume of the space region in the 1st octant bounded by the coordinate planes ($x=0$, $y=0$, $z=0$) and the surface $z=4-x^2-y$.



Surface $z=4-x^2-y$ lies above R

$z=0 \Rightarrow y=4-x^2$

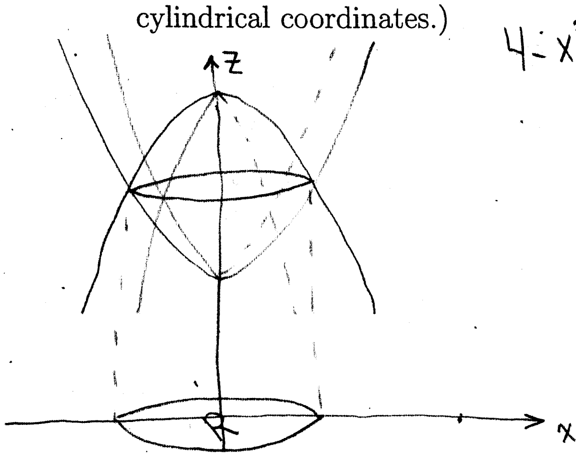
$$\text{Volume} = \iint_R \int_{z=0}^{4-x^2-y} 1\,dz\,dA$$

$$= \int_{x=0}^{x=2}\int_{y=0}^{y=4-x^2}\int_{z=0}^{z=4-x^2-y} 1\,dz\,dy\,dx \xrightarrow{\text{OVER}}$$

3. (4 points) A solid in space is bounded above by the paraboloid $z=4-x^2-y^2$ and below by the paraboloid $z=2+x^2+y^2$. Its density at the point (x,y,z) is given by $\rho(x,y,z)=1+x^2+y^2$. Use a triple integral to find the mass of the solid. (Hint: Use cylindrical coordinates.)

$4-x^2-y^2=2+x^2+y^2$

$x^2+y^2=1$



$x^2+y^2=1$

$$\text{Mass} = \iint_R \int_{z=2+x^2+y^2}^{z=4-x^2-y^2} (1+x^2+y^2)\,dz\,dA$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=1}\int_{z=2+r^2}^{z=4-r^2} (1+r^2)\,r\,dz\,dr\,d\theta$$

$\xrightarrow{\text{OVER}}$

#1 CONTINUED

$$\pi \int_1^2 \ln u \, du = \pi\left[u \ln u \Big|_1^2 - \int_1^2 \frac{u}{u}\, du \right] = \boxed{\pi \; 2\ln 2 - 1}$$

$v = \ln u \qquad dv = \frac{1}{u}\,du$

$dw = du \qquad w = u$

#2 CONTINUED

$$\int_0^2 \int_0^{4-x^2} (4-x^2-y)\, dy\, dx = \int_0^2 4y - x^2y - \frac{1}{2}y^2 \Big|_0^{4-x^2} dx$$

$$= \int_0^2 \left(16 - 4x^2 - 4x^2 + x^4 - 8 + 4x^2 - \frac{1}{2}x^4\right) dx = \int_0^2 \left(8 - 4x^2 + \frac{1}{2}x^4\right) dx$$

$$= 8x - \frac{4}{3}x^3 + \frac{1}{10}x^5 \Big|_0^2 = 16 - \frac{32}{3} + \frac{32}{10}$$

$$= \boxed{\frac{128}{15}}$$

#3 CONTINUED

$$2\pi \int_0^1 (1+r^2)\, r\, z \Big|_{z=2+r^2}^{z=4-r^2} dr = 2\pi \int_0^1 r(1+r^2)(2-2r^2)\, dr$$

$$= 2\pi \int_0^1 (2r - 2r^5)\, dr$$

$$= 2\pi \left(1 - \frac{1}{3}\right) = \boxed{\frac{4\pi}{3}}$$