Math 233 - Quiz 4

February 19, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3.5 points) Find $\vec{r}(t)$ if $\vec{r}\,'(t) = -8\cos(4t)\,\hat{\imath} + te^{-t^2}\,\hat{\jmath} + \dfrac{3}{t^2+1}\,\hat{k}$ and $\vec{r}(0) = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$.

$u = -t^2 \quad du = -2t\,dt$

$\vec{r}(t) = \left(-2\sin 4t + C_1\right)\hat{\imath} + \left(-\frac{1}{2}e^{-t^2} + C_2\right)\hat{\jmath} + \left(3\tan^{-1} t + C_3\right)\hat{k}$

$\vec{r}(0) = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k} \Rightarrow C_1 = 4,\ -\frac{1}{2} + C_2 = 3,\ C_3 = -2$

$C_2 = \frac{7}{2}$

$$\vec{r}(t) = \left(-2\sin 4t + 4\right)\hat{\imath} + \left(-\frac{1}{2}e^{-t^2} + \frac{7}{2}\right)\hat{\jmath} + \left(3\tan^{-1} t - 2\right)\hat{k}$$

2. (3.5 points) For $t > 0$, let $\vec{r}(t) = (\cos t + t\sin t)\hat{\imath} + (\sin t - t\cos t)\hat{\jmath}$. Compute the unit tangent vector, $T(t)$.

$\vec{r}\,'(t) = \left(-\sin t + \sin t + t\cos t\right)\hat{\imath} + \left(\cos t - \cos t + t\sin t\right)\hat{\jmath}$

$= t\cos t\,\hat{\imath} + t\sin t\,\hat{\jmath}$

$\|\vec{r}\,'(t)\| = \sqrt{t^2\cos^2 t + t^2\sin^2 t} = \sqrt{t^2} = t, \quad t \geq 0$

$$\hat{T}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$$

3. (3 points) Let $\vec{r}(t) = t^2\,\hat{\imath} + t^3\,\hat{\jmath} + t\,\hat{k}$. Set up the definite integral that gives the length of the graph of $\vec{r}$ from the point $(0,0,0)$ to the point $(4,8,2)$. Use your calculator to approximate the value of your integral. ($t=0$ at $(0,0,0)$; $t=2$ at $(4,8,2)$)

$\vec{r}\,'(t) = 2t\,\hat{\imath} + 3t^2\,\hat{\jmath} + \hat{k}$

$\|\vec{r}\,'(t)\| = \sqrt{4t^2 + 9t^4 + 1}$

$$S = \int_0^2 \sqrt{4t^2 + 9t^4 + 1}\,dt \approx 9.57$$