Math 233 - Quiz 8

April 2, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Let $g(x,y,z) = \sqrt{3x^2+y^2-2z^2}$. Compute $g_z(1,-2,1)$.

$$g_z(x,y,z) = \frac{1}{2}\left(3x^2+y^2-2z^2\right)^{-1/2}\cdot(-4z)$$

$$= \frac{-2z}{\sqrt{3x^2+y^2-2z^2}} \quad \Rightarrow \quad g_z(1,-2,1) = \boxed{\frac{-2}{\sqrt{5}}}$$

2. (2 points) Suppose you are given a function $f(x,y)$. State the conditions under which we should expect that $f_{xy}(x,y) = f_{yx}(x,y)$.

OUR THEOREM SAID ...

IF f_{xy} AND f_{yx} ARE CONTINUOUS IN A NEIGHBORHOOD OF (x,y),

THEN $f_{xy}(x,y) = f_{yx}(x,y)$.

3. (5 points) The volume V of a right circular cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. Suppose that the height decreases from 20 in to 19.95 in, while the radius increases from 4 in to 4.06 in. Find the total differential, dV. Then use differentials to approximate the change in volume.

$$dV = \frac{2}{3}\pi rh\,dr + \frac{1}{3}\pi r^2\,dh$$

$$\Delta V \approx \frac{2}{3}\pi rh\,\Delta r + \frac{1}{3}\pi r^2\,\Delta h$$

$h = 20$ in

$r = 4$ in

$\Delta h = -0.05$ in

$\Delta r = 0.06$ in

$$\Delta V \approx \frac{2}{3}\pi(4)(20)(0.06) + \frac{1}{3}\pi(4)^2(-0.05)$$

$$\approx 9.215338$$

$$\boxed{\Delta V \approx 9.2 \text{ in}^3}$$