Math 233 - Quiz 9

April 9, 2026

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (3 points) Use an appropriate linearization to approximate $e^{0.1} + \sin(3.1)$.

$f(x,y) = e^x + \sin y$ AT $(0,\pi)$...

$f(0,\pi) = 1$

$f_x(x,y) = e^x$ $\quad f_y(x,y) = \cos y$

$f_x(0,\pi) = 1$ $\quad f_y(0,\pi) = -1$

$L(x,y) = 1 + 1(x-0) + (-1)(y-\pi)$

$L(x,y) = 1 + x - (y-\pi)$

$L(0.1, 3.1) = 1 + 0.1 - (3.1 - \pi)$

≈ 1.142

2. (3 points) Find an equation of the plane tangent to the graph of $z = \tan^{-1}(xy)$ at the point where $x = 1/2$ and $y = 2$.

$z\Big|_{(\frac{1}{2},2)} = \tan^{-1}(1) = \frac{\pi}{4}$

$\frac{\partial z}{\partial x} = \frac{y}{(xy)^2+1}$ $\quad \frac{\partial z}{\partial x}\Big|_{(\frac{1}{2},2)} = \frac{2}{2} = 1$

$\frac{\partial z}{\partial y} = \frac{x}{(xy)^2+1}$ $\quad \frac{\partial z}{\partial y}\Big|_{(\frac{1}{2},2)} = \frac{1/2}{2} = \frac{1}{4}$

LINEARIZATION IS

$L(x,y) = \frac{\pi}{4} + (x - \frac{1}{2}) + \frac{1}{4}(y-2)$

TANGENT PLANE

$(x - \frac{1}{2}) + \frac{1}{4}(y-2) - (z - \frac{\pi}{4}) = 0$

3. (2 points) Suppose that w is a differentiable function of ~~x, y,~~ and ~~z~~. Further suppose that x, y, and z are differentiable functions of s and t. Write the chain rule formula for $\partial w/\partial s$.

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

4. (2 points) Assume that y is implicitly defined as a function of x by the equation $x^3 - 4xy + 2y^3 = x^2 + y$. Use partial derivatives to find dy/dx.

$F(x,y) = x^3 - 4xy + 2y^3 - x^2 - y$

$$\frac{dy}{dx} = \frac{-F_x}{F_y} = \frac{-(3x^2 - 4y - 2x)}{-4x + 6y^2 - 1}$$