Math 233 - Test 1
February 12, 2026

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) In this problem, the force vectors $\vec{F_1}$ and $\vec{F_2}$ are 2D vectors in the xy-plane.

   (a) The force $\vec{F_1}$ has magnitude 30 and has the same direction as the vector $\underbrace{-\hat{\imath}+\hat{\jmath}}$. Find the component form of $\vec{F_1}$.

   MAGNITUDE IS $\sqrt{(-1)^2+(1)^2} = \sqrt{2}$.

   $$\vec{F}_1 = \frac{30(-\hat{\imath}+\hat{\jmath})}{\sqrt{2}} = \boxed{-15\sqrt{2}\,\hat{\imath} + 15\sqrt{2}\,\hat{\jmath}}$$

   (b) The force $\vec{F_2}$ has magnitude 75 and makes a 150° angle with the positive x-axis. Find the component form of $\vec{F_2}$.

   $$\vec{F}_2 = 75\cos 150^\circ\,\hat{\imath} + 75\sin 150^\circ\,\hat{\jmath} = \boxed{-\frac{75\sqrt{3}}{2}\hat{\imath} + \frac{75}{2}\hat{\jmath}}$$

   (c) Refer to parts (a) and (b). Compute the resultant vector $\vec{F} = \vec{F_1} + \vec{F_2}$.

   $$\vec{F} = \left(-15\sqrt{2} - \frac{75\sqrt{3}}{2}\right)\hat{\imath} + \left(15\sqrt{2} + \frac{75}{2}\right)\hat{\jmath} \approx \boxed{-86.2\,\hat{\imath} + 58.7\,\hat{\jmath}}$$

   (d) Refer to part (c). What angle does $\vec{F}$ make with the positive x-axis?

   $\vec{F}$ IS IN QUAD II AND $\underbrace{\tan\theta = \dfrac{15\sqrt{2} + \frac{75}{2}}{-15\sqrt{2} - \frac{75\sqrt{3}}{2}}}$ $= -0.6814034626$

   $$\theta \approx -34.27^\circ + 180^\circ = \boxed{145.73^\circ}$$

2. (5 points) The vector $\vec{u}$ has terminal point $Z(5,-9)$ and initial point $A(6,-12)$. and the vector $\vec{w}$ has component form $\langle 4, 6\rangle$. Compute $\|3\vec{u} - 2\vec{w}\|$.

$\vec{u} = \vec{AZ} = -\hat{\imath} + 3\hat{\jmath}$

$\vec{w} = 4\hat{\imath} + 6\hat{\jmath}$

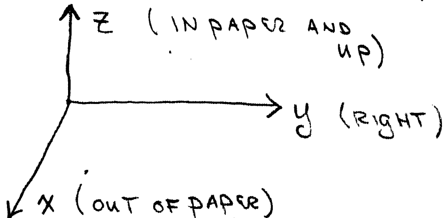
$$3\vec{u} - 2\vec{w} = -3\hat{\imath} + 9\hat{\jmath} - (8\hat{\imath} + 12\hat{\jmath})$$

$$-11\hat{\imath} - 3\hat{\jmath}$$

$$\|3\vec{u} - 2\vec{w}\| = \sqrt{(-11)^2 + (-3)^2} = \boxed{\sqrt{130} \approx 11.4}$$

3. (5 points) Explain how the right-hand rule gives the orientation of the coordinate axes in a 3-dimensional rectangular coordinate system. Then sketch and label a set of axes that are oriented according to your right-hand rule.

ON YOUR RIGHT HAND, POINT YOUR FINGERS IN THE DIRECTION OF POS. X-AXIS AND CURL YOUR FINGERS TOWARD THE POS. Y-AXIS. YOUR THUMB POINTS IN DIRECTION OF POS. Z-AXIS.



4. (6 points) Find the midpoint, M, of the line segment connecting $P\,(-2,4,-3)$ to $Q\,(-6,8,7)$. Then show that $\|\vec{PM}\| = \frac{1}{2}\|\vec{PQ}\|$.

$$m = \left(\frac{-2+(-6)}{2}, \frac{4+8}{2}, \frac{-3+7}{2}\right)$$

$$\boxed{m = (-4, 6, 2)}$$

$\vec{PM} = \langle -2, 2, 5 \rangle$

$\vec{PQ} = \langle -4, 4, 10 \rangle$

NOTICE THAT $\vec{PQ} = 2\,\vec{PM}$

$\|\vec{PM}\| = \sqrt{4+4+25} = \sqrt{33}$

$\|\vec{PQ}\| = \sqrt{16+16+100} = \sqrt{132} = 2\sqrt{33}$

$\|\vec{PM}\| = \frac{1}{2}\|\vec{PQ}\|$

5. (6 points) Let $\vec{x} = 3\hat{\imath} + 2\hat{\jmath} - 7\hat{k}$.

(a) Find a vector, different from $\vec{x}$, that is parallel to $\vec{x}$. Give a one-sentence explanation for how you know.

"PARALLEL TO" MEANS A "NONZERO SCALAR MULTIPE OF".

$2\vec{x} = \boxed{6\hat{\imath} + 4\hat{\jmath} - 14\hat{k}}$ IS, THEREFORE, PARALLEL TO $\vec{x}$

(b) Find a nonzero vector that is orthogonal to $\vec{x}$. Give a one-sentence explanation for how you know.

LET $\boxed{\vec{w} = \hat{\imath} + 9\hat{\jmath} + 3\hat{k}.}$

$\vec{w}$ IS ORTHOGONAL TO $\vec{x}$ BECAUSE $\vec{x} \cdot \vec{w} = 0$

$$\vec{x} \cdot \vec{w} = 3(1) + 2(9) - 7(3) = 0$$

6. (6 points) Suppose that θ is the angle between the two nonzero vectors $\vec{x}$ and $\vec{y}$. What can you say about $\vec{x} \cdot \vec{y}$ in each of these cases?

   (a) θ is an obtuse angle. $\vec{x} \cdot \vec{y} < 0$ (NEGATIVE)

   (b) θ is a right angle. $\vec{x} \cdot \vec{y} = 0$

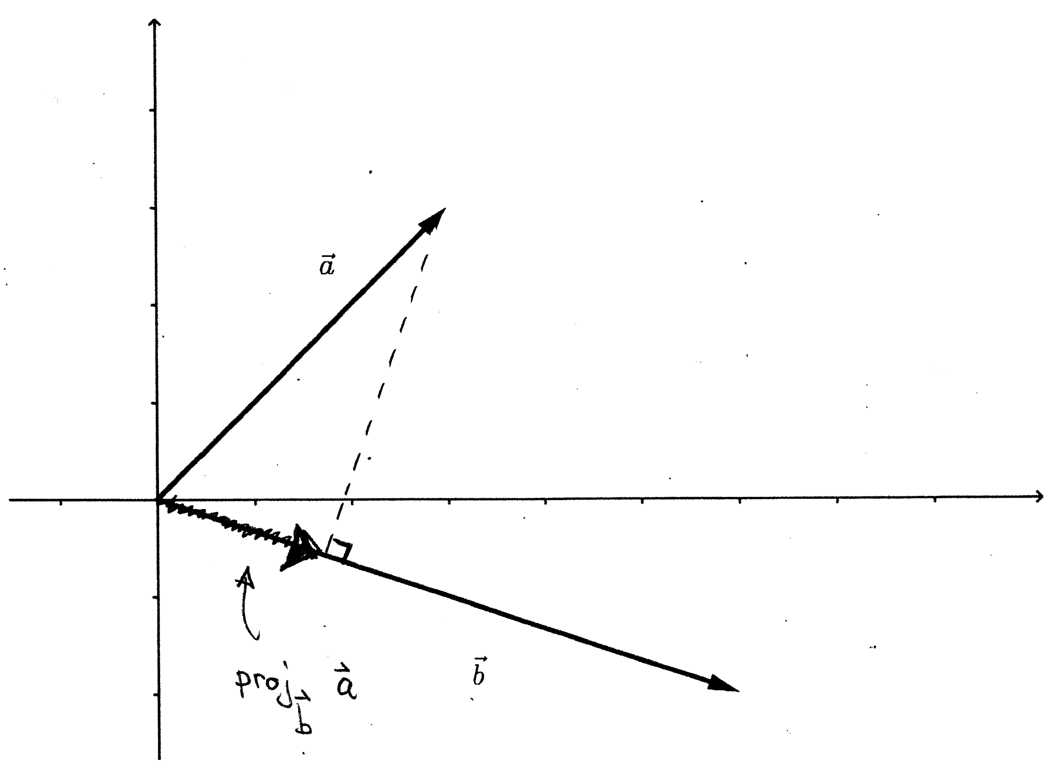
   (c) θ is an acute angle. $\vec{x} \cdot \vec{y} > 0$ (POSITIVE)

7. (4 points) Find the projection of $\vec{s} = -3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ onto $\vec{t} = -3\hat{\imath} - \hat{\jmath} + 5\hat{k}$.

$$\text{proj}_{\vec{t}}\, \vec{s} = \frac{\vec{s} \cdot \vec{t}}{\vec{t} \cdot \vec{t}}\, \vec{t} = \frac{9-2-20}{9+1+25}\left(-3\hat{\imath} - \hat{\jmath} + 5\hat{k}\right)$$

$$= \frac{-13}{35}\left(-3\hat{\imath} - \hat{\jmath} + 5\hat{k}\right) = \boxed{\frac{39}{35}\hat{\imath} + \frac{13}{35}\hat{\jmath} - \frac{65}{35}\hat{k}}$$

8. (4 points) The figure below shows the vectors $\vec{a}$ and $\vec{b}$. Sketch $\text{proj}_{\vec{b}}\, \vec{a}$.



9. (8 points) Find the measure of the angle between the planes described by the equations below. Write your final answer in degrees rounded to the nearest hundredth.

$$2x - y + 2z = 7 \qquad\qquad -5x + 3z = 12$$

$\vec{n}_1 = 2\hat{i} - \hat{j} + 2\hat{k}$ $\qquad$ $\vec{n}_2 = -5\hat{i} + 3\hat{k}$

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\|\|\vec{n}_2\|} = \frac{|-4|}{\sqrt{9}\sqrt{34}} = \frac{4}{3\sqrt{34}} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{3\sqrt{34}}\right) \approx \boxed{76.78^\circ}$$

10. (10 points) The distance from a point Q to the line passing through P and parallel to $\vec{v}$ is given by

$$D = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|}.$$

Use this formula below.

(a) Choose any point on the line described by the parametric equations below. Let your point be Q. (There are infinitely many choices for Q.)

$$x = 3t - 4, \quad y = -5t, \quad z = t + 5.$$

WE CAN READ FROM EQUATIONS $(t=0)$

$\boxed{Q(-4,0,5)}$

(b) Now consider the line ℓ with symmetric equations

$$\frac{x+6}{2} = y - 3 = \frac{z-1}{-3}.$$

Find a point P on ℓ and a vector $\vec{v}$ parallel to ℓ.

WE CAN READ THESE RIGHT OFF :

$P(-6,3,1)$

$\vec{v} = 2\hat{i} + \hat{j} - 3\hat{k}$

(c) Compute the distance from Q to the line ℓ.

$\vec{PQ} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{PQ} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 2 & 1 & -3 \end{vmatrix}$$

$= 5\hat{i} + 14\hat{j} + 8\hat{k}$

$\|\vec{PQ} \times \vec{v}\| = \sqrt{25 + 196 + 64} = \sqrt{285}$

$\|\vec{v}\| = \sqrt{4+1+9} = \sqrt{14}$

$\text{DISTANCE} = \sqrt{\frac{285}{14}} \approx 4.51$

11. (10 points) Find an equation of the plane passing through the points $R(1,-2,4)$, $S(0,3,-5)$, and $T(8,2,-3)$.

$$\vec{RS} = -\hat{\imath} + 5\hat{\jmath} - 9\hat{k}$$
$$\vec{RT} = 7\hat{\imath} + 4\hat{\jmath} - 7\hat{k}$$

$$\vec{n} = \vec{RS} \times \vec{RT} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 5 & -9 \\ 7 & 4 & -7 \end{vmatrix}$$

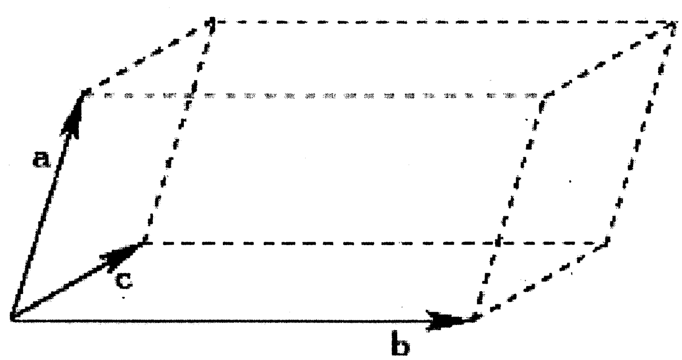
$$= \hat{\imath} - 70\hat{\jmath} - 39\hat{k}$$

Using $S(0,3,-5)$, PLANE IS

$$x - 70y - 39z = (0) - 70(3) - 39(-5) = -15$$

$$\boxed{x - 70y - 39z = -15}$$

12. (8 points) A crystal structure has the form of a parallelepiped determined by the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = 3\hat{\jmath} + 5\hat{k}$, and $\vec{c} = -4\hat{\imath} + 2\hat{\jmath} + \hat{k}$, where distances are measured in micrometers. Find the volume of the parallelepiped.



$$\vec{a} \cdot (\vec{b} \times \vec{c})$$

$$= \begin{vmatrix} 1 & 2 & 1 \\ 0 & 3 & 5 \\ -4 & 2 & 1 \end{vmatrix} = (1)(-7) - (2)(20) + (1)(12)$$

$$= -35$$

$$\boxed{\text{Volume} = 35\,\mu m^3}$$

13. (4 points) Find a set of parametric equations for the line through $A(4,-2,3)$ and $B(0,-2,8)$.

$\vec{AB} = -4\hat{\imath} + 0\hat{\jmath} + 5\hat{k}$

Using $A(4,-2,3)$ ...

$$\boxed{\begin{aligned} x &= -4t+4 \\ y &= -2 \\ z &= 5t+3 \end{aligned}}$$

Notice that $t=0$ gives A and $t=1$ gives B.

14. (6 points) Let $\vec{r}(t) = \dfrac{\sin t}{t}\hat{\imath} + \ln(t+1)\hat{\jmath} + e^{2t}\hat{k}$.

(a) Determine the domain of $\vec{r}$.

Must have $t \neq 0$ and $t+1 > 0$

$$\boxed{\text{Domain} = \{t : t \neq 0 \text{ and } t > -1\}}$$

(b) Compute $\lim_{t\to 0} \vec{r}(t)$.

$$\Big(\underbrace{\lim_{t\to 0} \frac{\sin t}{t}}_{1}\Big)\hat{\imath} + \Big(\underbrace{\lim_{t\to 0} \ln(t+1)}_{0}\Big)\hat{\jmath} + \Big(\underbrace{\lim_{t\to 0} e^{2t}}_{1}\Big)\hat{k} = \boxed{\hat{\imath} + \hat{k}}$$

15. (8 points) Let $\vec{r}(t) = 2\cos t\,\hat{\imath} - 2\sin t\,\hat{\jmath} - 3\,\hat{k}$.

(a) Compute $\|\vec{r}(t)\|$.

$$\|\vec{r}(t)\| = \sqrt{\underbrace{4\cos^2 t + 4\sin^2 t}_{4(\cos^2 t + \sin^2 t)} + 9} = \sqrt{4+9} = \boxed{\sqrt{13}}$$

(b) Determine the derivative $\vec{r}\,'(t)$.

$$\boxed{\vec{r}\,'(t) = -2\sin t\,\hat{\imath} - 2\cos t\,\hat{\jmath}}$$

(c) Compute $\vec{r}(t)\cdot\vec{r}\,'(t)$. $= \boxed{0}$

$\vec{r}$ has const. mag. It's orthog. to its derivative.

(d) Compute $\vec{r}(t) \times \vec{r}\,'(t)$.

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2\cos t & -2\sin t & -3 \\ -2\sin t & -2\cos t & 0 \end{vmatrix} = \hat{\imath}(-6\cos t) - \hat{\jmath}(-6\sin t) + \hat{k}(-4)$$

$$= \boxed{-6\cos t\,\hat{\imath} + 6\sin t\,\hat{\jmath} - 4\hat{k}}$$