Math 233 - Test 1

February 12, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) In this problem, the force vectors $\vec{F_1}$ and $\vec{F_2}$ are 2D vectors in the xy-plane.

   (a) The force $\vec{F_1}$ has magnitude 30 and has the same direction as the vector $-\hat{\imath} + \hat{\jmath}$. Find the component form of $\vec{F_1}$.

   (b) The force $\vec{F_2}$ has magnitude 75 and makes a 150° angle with the positive x-axis. Find the component form of $\vec{F_2}$.

   (c) Refer to parts (a) and (b). Compute the resultant vector $\vec{F} = \vec{F_1} + \vec{F_2}$.

   (d) Refer to part (c). What angle does $\vec{F}$ make with the positive x-axis?

2. (5 points) The vector $\vec{u}$ has terminal point $Z(5, -9)$ and initial point $A(6, -12)$. and the vector $\vec{w}$ has component form $\langle 4, 6 \rangle$. Compute $\|3\vec{u} - 2\vec{w}\|$.

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3. (5 points) Explain how the right-hand rule gives the orientation of the coordinate axes in a 3-dimensional rectangular coordinate system. Then sketch and label a set of axes that are oriented according to your right-hand rule.

4. (6 points) Find the midpoint, M, of the line segment connecting P $(-2, 4, -3)$ to Q $(-6, 8, 7)$. Then show that $\|\vec{PM}\| = \frac{1}{2}\|\vec{PQ}\|$.

5. (6 points) Let $\vec{x} = 3\hat{\imath} + 2\hat{\jmath} - 7\hat{k}$.

   (a) Find a vector, different from $\vec{x}$, that is parallel to $\vec{x}$. Give a one-sentence explanation for how you know.

   (b) Find a nonzero vector that is orthogonal to $\vec{x}$. Give a one-sentence explanation for how you know.

6. (6 points) Suppose that θ is the angle between the two nonzero vectors $\vec{x}$ and $\vec{y}$. What can you say about $\vec{x} \cdot \vec{y}$ in each of these cases?
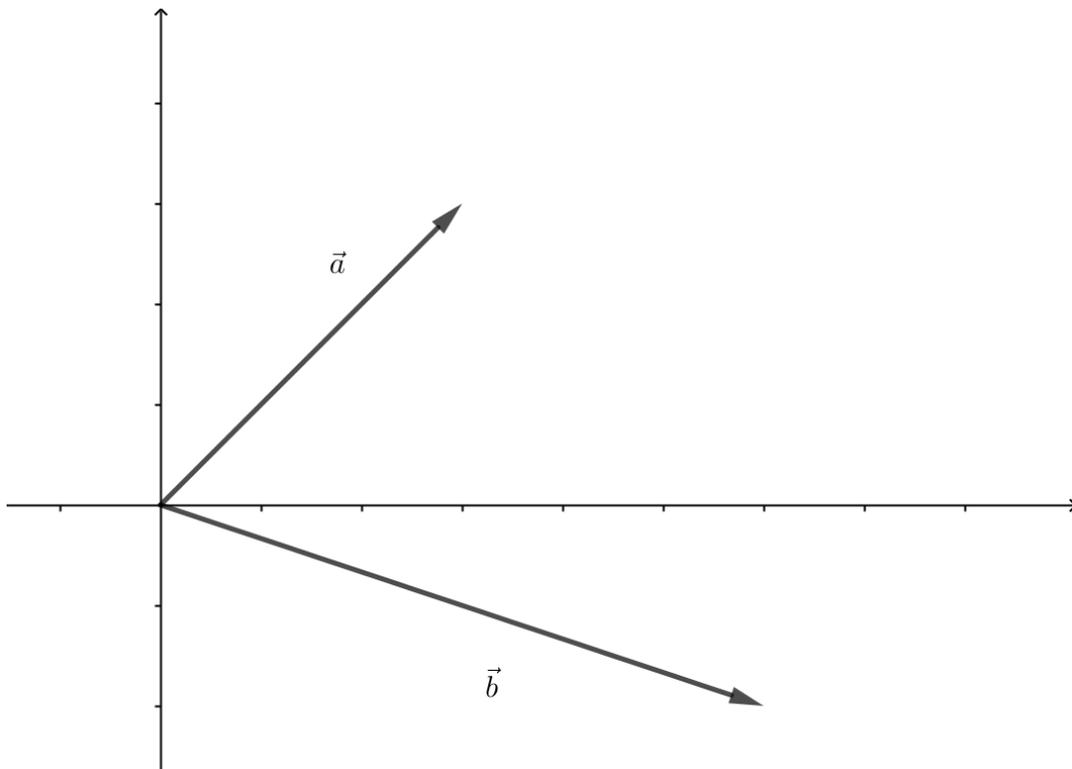
   (a) θ is an obtuse angle.

   (b) θ is a right angle.

   (c) θ is an acute angle.

7. (4 points) Find the projection of $\vec{s} = -3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ onto $\vec{t} = -3\hat{\imath} - \hat{\jmath} + 5\hat{k}$.

8. (4 points) The figure below shows the vectors $\vec{a}$ and $\vec{b}$. Sketch $\operatorname{proj}_{\vec{b}} \vec{a}$.

9. (8 points) Find the measure of the angle between the planes described by the equations below. Write your final answer in degrees rounded to the nearest hundredth.

$$2x - y + 2z = 7 \qquad\qquad -5x + 3z = 12$$

10. (10 points) The distance from a point Q to the line passing through P and parallel to $\vec{v}$ is given by

$$D = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|}.$$

Use this formula below.

(a) Choose any point on the line described by the parametric equations below. Let your point be Q. (There are infinitely many choices for Q.)

$$x = 3t - 4, \quad y = -5t, \quad z = t + 5.$$

(b) Now consider the line ℓ with symmetric equations

$$\frac{x+6}{2} = y - 3 = \frac{z-1}{-3}.$$

Find a point P on ℓ and a vector $\vec{v}$ parallel to ℓ.

(c) Compute the distance from Q to the line ℓ.

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11. (10 points) Find an equation of the plane passing through the points $R(1, -2, 4)$, $S(0, 3, -5)$, and $T(8, 2, -3)$.

12. (8 points) A crystal structure has the form of a parallelepiped determined by the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = 3\hat{\jmath} + 5\hat{k}$, and $\vec{c} = -4\hat{\imath} + 2\hat{\jmath} + \hat{k}$, where distances are measured in micrometers. Find the volume of the parallelepiped.
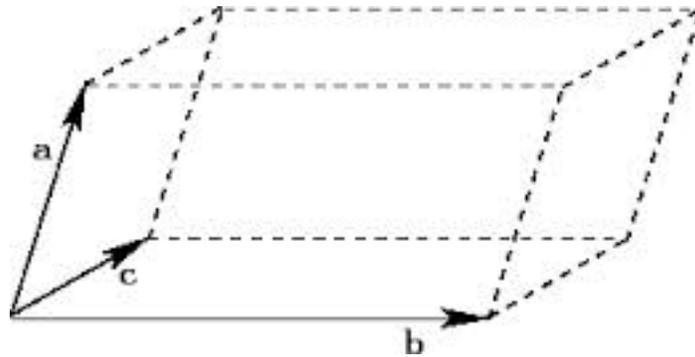
13. (4 points) Find a set of parametric equations for the line through $A\,(4, -2, 3)$ and $B\,(0, -2, 8)$.

14. (6 points) Let $\vec{r}(t) = \dfrac{\sin t}{t}\hat{\imath} + \ln(t+1)\hat{\jmath} + e^{2t}\hat{k}$.

    (a) Determine the domain of $\vec{r}$.

    (b) Compute $\lim\limits_{t \to 0} \vec{r}(t)$.

15. (8 points) Let $\vec{r}(t) = 2\cos t\,\hat{\imath} - 2\sin t\,\hat{\jmath} - 3\,\hat{k}$.

    (a) Compute $\|\vec{r}(t)\|$.

    (b) Determine the derivative $\vec{r}\,'(t)$.

    (c) Compute $\vec{r}(t) \cdot \vec{r}\,'(t)$.

    (d) Compute $\vec{r}(t) \times \vec{r}\,'(t)$.