Math 233 - Test 2

March 12, 2026

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Find the function $\vec{r}(t)$ that satifies the following conditions:

$$\vec{r}\,'(t) = te^{-t^2}\,\hat{\imath} + \cos(2t)\,\hat{\jmath} + \frac{1}{1+t^2}\,\hat{k}; \quad \vec{r}(0) = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}.$$

2. (8 points) The graph of the $\vec{r}(t) = (t + \sin t)\,\hat{\imath} + (1 - \cos t)\,\hat{\jmath}$ is an example of a curve called a *cycloid*. Find the arc length of the cycloid from $t = 0$ to $t = \pi$. (Your integral should be easy if you use the trig identity $1 + \cos t = 2\cos^2(t/2)$.)

3. (10 points) Let $\vec{r}(t) = 2t\,\hat{\imath} + \cos(3t)\,\hat{\jmath} - \sin(3t)\hat{k}$. Starting from $t = 0$, find the arc-length parameter $s(t)$, and then reparameterize $\vec{r}$ in terms of s.

   Follow-up question: What is the magnitude of the derivative of your reparameterized function?

4. (8 points) Let $\vec{r}(t) = (t^3 - 3t)\,\hat{\imath} + (3t - 5)^2\,\hat{\jmath} - 2t\hat{k}$. Compute $\hat{T}(2)$ and confirm that it is a unit vector.

5. (8 points) A projectile is launched from 6 feet above the ground at an angle of $12°$ with the horizontal. The projectile is to have a range of 250 feet. Find the initial velocity. **To get credit for this problem, you must set up and use the vector-valued function that gives the position of the projectile at time t.** (Use $g = 32\,\text{ft}/\text{sec}^2$.)

6. (8 points) For $n \geq 2$, think about the graph of $y = x^n$. Compute the curvature at $x = 1$. Then show that $\lim_{n\to\infty} \kappa(1) = 0$.

7. (4 points) An object is moving along a line and its acceleration vector is given by

$$\vec{a}(t) = \frac{e^t\,\hat{\imath} + te^t\,\hat{\jmath}}{\sqrt{1+t^2}}.$$

Assume that $\vec{a}(t)$ has the same direction as $\hat{T}(t)$ and find the tangential component of the acceleration, a_T.

8. (6 points) Imagine you are moving at a constant (nonzero) speed.

   (a) Explain how it is possible that you may be accelerating.

   (b) Assuming that you are accelerating, describe the relationship between your velocity and acceleration vectors.

9. (10 points) Compute $\hat{N}(0)$ when $\vec{r}(t) = \sqrt{2}t\,\hat{\imath} + e^t\,\hat{\jmath} + e^{-t}\,\hat{k}$.

10. (10 points) Each of these equations defines a surface in 3-space. Briefly describe each surface.

   (a) $10 + 3x^2 - 8y^2 + 5z^2 = 0$

   (b) $4x^2 - \frac{1}{4}y^2 - \frac{1}{9}z^2 = -1$

   (c) $x = (z+1)^2 + 1$

   (d) $4x^2 + 25y^2 - z = 1$

   (e) $y = 4x^2 - z^2$

11. (6 points) Let $f(x,y) = \frac{\sqrt{x^2+y^2-9}}{x}$.

   (a) What is the domain of f?

   (b) Sketch the level curve $f(x,y) = 0$.

   (c) Describe the level curve $f(x,y) = 2$.

item (6 points) Let $g(x, y, z) = \dfrac{x - y + z}{2x + y - z}$.

(a) Evaluate $g(1, 0, -2)$.

(b) What is the domain of g?

(c) Describe the level surface $g(x, y, z) = 2$.

12. (10 points) Find each limit or show that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(2,1)} \frac{x - y - 1}{\sqrt{x - y} - 1}$

(b) $\displaystyle\lim_{(x,y)\to(3,3)} \frac{x^4 - y^4}{x^2 - y^2}$