

Math 233 - Final Exam B

May 14, 2026

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) Let $\vec{r}(t) = \frac{1}{3}t^3 \hat{i} + 2t^2 \hat{j}$ for $t \geq 0$.

(a) Find the arc-length parameter, $s(t)$, starting from $t = 0$.

(b) Use your arc length parameter to find the length of the curve from $t = 0$ to $t = 3$.

2. (10 points) Find and classify the critical points of $g(x, y) = x^2 + 2y^2 - x^2y$. Identify all relative extreme values and saddle points.

3. (10 points) Consider the surface described by the equation $z = 2e^{4x^2+2xy-4y}$.

(a) Find an equation of the plane tangent to the surface at the point $(1, 2, 2)$.

(b) Find a set of parametric equations for the line normal to the surface at the point $(1, 2, 2)$.

4. (10 points) Use Lagrange multipliers to find the minimum and maximum values of $f(x, y) = xy$ on the circle $x^2 + y^2 - 8 = 0$.

5. (10 points) Consider the double integral

$$\iint_R \sin(y^3) dA,$$

where R is the 1st quadrant region enclosed by the graphs of $x = 0$, $y = 2$, and $y = \sqrt{x}$. Sketch the region of integration, write the double integral as an iterated integral with both orders of integration, and evaluate either one of your iterated integrals.

6. (10 points) Use a double integral in polar coordinates to find the volume of the solid bounded above by the cone $z = \sqrt{x^2 + y^2}$, bounded below by the xy -plane, and lying inside the cylinder $x^2 + y^2 = 2y$. (Some helpful information is given below.)

These ideas may be useful...

- Complete the square to find the center and radius of the circle $x^2 + y^2 = 2y$.
- In polar coordinates, the circle $x^2 + y^2 = 2y$ has equation $r = 2 \sin \theta$.
- $\int \sin^3 \theta \, d\theta = \frac{1}{3} \cos^3 \theta - \cos \theta + C$

7. (10 points) Use a triple integral to find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + y + 4z = 4$.