Math 236 - Assignment 10

April 17, 2024

Name ____________________

Score ______________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 24.

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1. In using the *method of variation of parameters* to solve a differential equation, one must solve the system
$$\begin{aligned} y_1(x)v_1'(x) + y_2(x)v_2'(x) &= 0 \\ y_1'(x)v_1'(x) + y_2'(x)v_2'(x) &= g(x) \end{aligned}$$
for $v_1'(x)$ and $v_2'(x)$, where y_1, y_2, and g are known functions. Use Cramer's rule to solve the system.

2. Prove that for square matrices "is similar to" is an equivalence relation.

3. Show by computation that $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is not diagonalizable.

4. Let $\begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$. Find the characteristic polynomial. Show that 2 is an eigenvalue of A. Find a basis for the eigenspace corresponding to $\lambda = 2$.

5. Find the characteristic polynomial of A.
$$\begin{pmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

6. Construct a 2×2 matrix with only one (distinct) eigenvalue.

7. Show that if A^2 is the zero matrix, then the only eigenvalue of A is 0.

8. Diagonalize the following matrix.
$$\begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}$$