## Math 236 - Assignment 1

January 17, 2024

Name $\qquad$
Score $\qquad$

Show all work to receive full credit. Supply explanations when necessary. Do all computations by hand. This assignment is due January 24.

1. Find the solution set by reducing to echelon form.

$$
\begin{aligned}
x_{1}-3 x_{2}+4 x_{3} & =-4 \\
3 x_{1}-7 x_{2}+7 x_{3} & =-8 \\
-4 x_{1}+6 x_{2}-x_{3} & =7
\end{aligned}
$$

2. Find the solution set by reducing to echelon form.

$$
\begin{aligned}
x_{1} & -3 x_{3}
\end{aligned}=8, \begin{aligned}
2 x_{1}+2 x_{2} & +9 x_{3}
\end{aligned}=70
$$

3. Prove that each elementary row operation in Gauss's method is reversible.
4. For which values of $b$ are there no solutions, infinitely many solutions, or a unique solution?

$$
\begin{aligned}
& 2 x+y=7 \\
& 8 x+4 y=b
\end{aligned}
$$

5. Consider the system shown below with variables $x$ and $y$. Use geometric reasoning to explain why there are three possibilities: no solution, infinitely many solutions, unique solution.

$$
\begin{aligned}
a x+b y & =c \\
d x+e y & =f
\end{aligned}
$$

6. Find the solution set by reducing to echelon form. Write the solution set in vector notation, identifying a particular solution and the solution of the corresponding homogeneous system.

$$
\begin{aligned}
x & -z \\
y+2 z-w & =3 \\
x+2 y+3 z-w & =7
\end{aligned}
$$

7. Find the coefficients $a, b$, and $c$ so that the graph of $p(x)=a x^{2}+b x+c$ passes through the points $(1,-6),(2,-9)$, and $(-1,-12)$.
8. Describe all functions $f(x)=a x^{2}+b x+c$ such that $f(1)=2$. Do so by writing $\left(\begin{array}{l}a \\ b \\ c\end{array}\right)=$ particular + homogeneous.
