

# Math 236 - Assignment 2

January 24, 2024

Name \_\_\_\_\_

Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations when necessary. Do all computations by hand unless otherwise indicated. This assignment is due January 31.

---

1. For any real numbers  $x$  and  $y$ , we will say  $x \sim y$  if and only if  $x - y$  is an integer. Prove that  $\sim$  is an equivalence relation.
2. Prove that a linear combination of three linear combinations of  $x$ ,  $y$ , and  $z$  is a linear combination of  $x$ ,  $y$ , and  $z$ .
3. Find (by hand) the reduced row echelon form (RREF).

$$\begin{pmatrix} 1 & 0 & 3 & 1 & 2 \\ 1 & 4 & 2 & 1 & 5 \\ 3 & 4 & 8 & 1 & 2 \end{pmatrix}$$

4. Write the system as an augmented matrix. Then compute the RREF and determine the solution set. You may use technology to find the RREF.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + x_4 - x_5 &= 1 \\ 3x_1 - x_2 + x_3 + x_4 + x_5 &= 3 \end{aligned}$$

5. Say why the following matrix is NOT in RREF. Then reduce it to RREF. Finally, once it's reduced, show that no row is a linear combination of the other rows.

$$\begin{pmatrix} 1 & 2 & 4 & 0 & 2 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

6. Give two distinct echelon forms of the matrix below. Be sure to say (or show) which sequence of row operations gave each form.

$$\begin{pmatrix} 2 & 1 & 1 & 3 \\ 6 & 4 & 1 & 2 \\ 1 & 5 & 1 & 5 \end{pmatrix}$$

7. Reduce to RREF. Then express the third row of the RREF as a linear combination of the rows of the original matrix.

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 0 \\ 3 & -1 & 0 \end{pmatrix}$$

8. Three truck drivers (in perhaps 1960) went into a roadside cafe. One truck driver purchased four sandwiches, a cup of coffee, and ten doughnuts for \$8.45. Another driver purchased three sandwiches, a cup of coffee, and seven doughnuts for \$6.30. What did the third truck driver pay for a sandwich, a cup of coffee, and a doughnut? (If your approach to this problem involves matrices and elimination, you may use technology.)