Math 236 - Assignment 3

January 31, 2024

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations when necessary. Do all computations by hand unless otherwise indicated. This assignment is due February 7.

1. Let V be the set of all vectors in $\mathbb{R}^3$ with the usual scalar multiplication. However, define addition '+' in V as follows:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_a \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 \\ z_1 \end{pmatrix}.$$

   Show that V is NOT a vector space.

2. Show that the set of all 2×2 diagonal matrices

$$\left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} : a, b \in \mathbb{R} \right\}$$

   with the usual operations of matrix addition and scalar multiplication is a vector space.

3. Show that the set of all differentiable functions (of a single variable) with the usual operations of function addition and multiplication by a real constant is a vector space.

4. Show that the set $\mathbb{R}^+$ of positive real numbers is a vector space when we interpret the "sum", $x + y$, as the product of x and y, and we interpret scalar "multiplication", $k \cdot x$, as the kth power of x.

5. Each element in a vector space must have an additive inverse. Prove that for each element x in vector space V, its additive inverse is unique. Use only the ten vector space conditions! (Hint: Let y and z be the additive inverses of x, and then show that y must be equal to z.)

6. Is this a subspace of P_2: $\{ax^2 + bx + c : a = 1\}$?

7. Determine if $\begin{pmatrix} 0 & 1 \\ 4 & 2 \end{pmatrix}$ is in the span of $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 2 & 0 \\ 2 & 3 \end{pmatrix}$. What about $\begin{pmatrix} -5 & 0 \\ -5 & -12 \end{pmatrix}$?

8. Parameterize the subspace's description. Then express the subspace as a span of vectors in $M_{2\times 2}$.

$$\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : 2a - c - d = 0 \text{ and } a + 3b = 0 \right\}$$

9. Suppose that U and W are subspaces of the vector space V. Prove that $U \cap W$ is a subspace of V. (Recall that '$\cap$' stands for the intersection. Every element in $U \cap W$ is in both U and W.)