Math 236 - Assignment 4

Name ____________________

February 14, 2024

Score __________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due February 21.

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1. Determine whether the set is a linearly dependent or independent subset of $\mathcal{P}_2$.

$$\{2 + x + 7x^2, 3 - x + 2x^2, 4 - 3x^2\}$$

2. Determine whether the set is a linearly dependent or independent subset of $M_{2\times 2}$.

$$\left\{ \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \right\}$$

3. Suppose that the set $\{\vec{u}, \vec{v}, \vec{w}\}$ is a linearly independent set. Prove that $\{\vec{u}, \vec{u} + \vec{v}, \vec{u} + \vec{v} + \vec{w}\}$ is also a linearly independent set.

4. Suppose that all proper subsets of $A = \{\vec{x}, \vec{y}, \vec{z}\}$ (except the empty set) are linearly independent. Must it be true that A itself is linearly independent?

5. Is this a basis for $\mathcal{P}_2$?

$$\{x^2 - x + 1, 2x + 1, 2x - 1\}$$

6. Represent $x + x^3$ with respect to the given basis for $\mathcal{P}_3$.

$$B = \{1, 1 + x, 1 + x + x^2, 1 + x + x^2 + x^3\}$$

7. Find a basis for the subspace below. Prove that it is a basis.

$$M = \{a + bx + cx^2 + dx^3 : 2a + b - c - 2d = 0\}$$

8. Find a basis for the vector space of symmetric 2×2 matrices.

9. Find a basis for the subspace of polynomials $p \in \mathcal{P}_3$ with $p(1) = 0$ and $p(2) = 0$. Prove that it is a basis.

10. Find a basis for, and the dimension of, the solution set of the following system.

$$\begin{array}{rcrcrcrcl} x_1 & - & 4x_2 & + & 3x_3 & - & x_4 & = & 0 \\ 2x_1 & - & 8x_2 & + & 6x_3 & - & 2x_4 & = & 0 \end{array}$$

11. What is the dimension of each vector space (or subspace) from problems 7–9?