Math 236 - Assignment 8

March 27, 2024

Name ____________________

Score ______________

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 3.

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1. Show that A is invertible for any θ and find A^{-1}.

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

2. For an invertible matrix A, prove that $(A^k)^{-1} = (A^{-1})^k$.

3. Argue that the inverse of any permutation matrix is its transpose. (You need not give a formal proof, just a compelling argument.)

4. Suppose that A and B are $n \times n$ matrices. Prove that $AB = I$ if and only if $BA = I$. (Helpful hint: From the fact that $\text{rank}(AB) = \text{rank}(BA) = n$, it follows that $\text{rank}(A) = \text{rank}(B) = n$.)

5. Find the change of basis matrix for $B, D \subseteq \mathbb{R}^2$.

$$B = \left\langle \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix} \right\rangle, \qquad D = \left\langle \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\rangle$$

6. Find the change of basis matrix for $B, D \subseteq \mathcal{P}_2$.

$$B = \langle 1, x, x^2 \rangle, \qquad D = \langle x^2, 1, x \rangle$$

7. Find bases such that this matrix represents the identity map with respect to those bases.

$$\begin{pmatrix} 3 & 1 & 4 \\ 2 & -1 & 1 \\ 0 & 0 & 4 \end{pmatrix}$$

8. Perform the Gram-Schmidt process on this basis for $\mathbb{R}^3$:

$$\left\langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \right\rangle.$$

9. Find an orthonormal basis for this subspace of $\mathbb{R}^4$:

$$\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} : x - y - z + w = 0 \text{ and } x + z = 0 \}.$$

10. What happens if we apply the Gram-Schmidt process to a finite set that is not linearly independent?

11. What happens if we apply the Gram-Schmidt process to a basis that is already orthogonal?