

Math 236 - Assignment 9

April 3, 2024

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. This assignment is due April 10.

1. Suppose that $B = \{\vec{\beta}_1, \vec{\beta}_2, \dots, \vec{\beta}_n\}$ is a set of nonzero mutually orthogonal vectors. Prove that the set is linearly independent.

2. Use any method to compute the determinant by hand.

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & -3 \\ 4 & 5 & 1 \end{pmatrix}$$

3. Use Gaussian elimination to compute the determinant by hand.

$$B = \begin{pmatrix} 1 & -1 & 1 & 2 \\ 1 & 0 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 1 & 1 & -1 & 1 \end{pmatrix}$$

4. Use Cramer's rule to solve the following system of equations.

$$\begin{aligned} 2x - 5y &= 7 \\ 4x + 9y &= 4 \end{aligned}$$

5. Which real numbers x make this matrix singular?

$$\begin{pmatrix} 12 - x & 4 \\ 8 & 8 - x \end{pmatrix}$$

6. Which real numbers θ make this matrix singular? Explain geometrically. (Perhaps a different problem on this assignment could be useful in thinking about a geometric explanation.)

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

7. For which values of k does this system have a unique solution?

$$\begin{aligned} x &+ z - w = 2 \\ &y - 2z = 3 \\ x &+ kz = 4 \\ &z - w = 2 \end{aligned}$$

8. What is the determinant of each kind of elementary matrix?

9. Use the Laplace expansion to compute the determinant by hand. Expand over whichever row or column is most convenient.

$$\begin{pmatrix} 1 & 5 & 0 \\ 2 & 1 & 1 \\ 3 & -1 & 0 \end{pmatrix}$$

10. Find the inverse by using the matrix adjoint.

$$\begin{pmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 1 & 8 & 9 \end{pmatrix}$$