Math 236 - Test 1

February 7, 2024

Name ______________________

Score __________

Show all work to receive full credit. Supply explanations when necessary. Unless otherwise indicated, you may use your calculator to obtain any RREF.

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1. (12 points) Reduce to echelon form (not RREF) and then backsolve by substitution. Do not use your calculator for the reduction. Indicate which row operations you used.

$$\begin{array}{rcrcrcr} x & + & y & - & z & = & -2 \\ 2x & - & 2y & + & 4z & = & 18 \\ x & & & - & 2z & = & -4 \end{array}$$

2. (8 points) Explain why the system is not in echelon form. Then reduce it to echelon form (not RREF) and state the leading and free variables. Do not solve the system, and do not use your calculator for the reduction.

$$\begin{array}{rcrcrcrcr} x & + & 2y & - & 3z & + & w & = & 8 \\ & & & & 6z & - & 2w & = & 8 \\ & & & - & 3z & + & w & = & -4 \end{array}$$

3. (9 points) Use your calculator to determine the RREF of the augmented matrix associated with each system. Write the RREF, and then simply say if the system has a unique solution, no solution, or infinitely many solutions.

(a)
$$\begin{array}{rcrcrcrcr} x & & & + & z & + & w & = & 4 \\ 2x & + & y & & & - & w & = & 2 \\ 3x & + & y & + & z & & & = & 7 \end{array}$$

(b)
$$\begin{array}{rcrcrcr} 2a & + & b & - & c & = & 2 \\ 2a & & & + & c & = & 3 \\ a & - & b & & & = & 0 \end{array}$$

(c)
$$\begin{array}{rcrcrcr} x_1 & & & + & x_3 & = & 4 \\ x_1 & - & x_2 & + & 2x_3 & = & 5 \\ 4x_1 & - & x_2 & + & 5x_3 & = & 17 \end{array}$$

4. (4 points) Say what it means for a matrix to be nonsingular. Then choose from any problem above (problems 1–3) an example of a nonsingular matrix.

5. (8 points) Write the system as an augmented matrix. Then compute the RREF and determine the complete solution set. You may use your calculator to find the RREF.

$$\begin{array}{rcrcrcrcrcr} 4x_1 & + & 12x_2 & + & x_3 & - & 2x_4 & - & 5x_5 & = & 17 \\ -x_1 & - & 3x_2 & & & + & x_4 & + & x_5 & = & -3 \\ 3x_1 & + & 9x_2 & + & x_3 & - & x_4 & - & 5x_5 & = & 16 \end{array}$$

6. (9 points) For any integers m and n, we will say $m \sim n$ if and only if $m - n$ is a multiple of 5. Prove that $\sim$ is an equivalence relation.

7. (3 points) For real numbers x and y, we will say $x \sim y$ if and only if x is the opposite of y. Prove that $\sim$ is NOT an equivalence relation.

8. (5 points) List all possible RREFs for a 2×2 matrix.

9. (5 points) Determine whether these matrices are row equivalent. If helpful, you may use your calculator, but explain your reasoning.

$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & -1 & 1 \\ 5 & -1 & 5 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 10 \\ 2 & 0 & 4 \end{pmatrix}$$

10. (10 points) The following set with the usual addition and scalar multiplication operations in $\mathbb{R}^3$ is a vector space.

$$W = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3 \ : \ a + 2b + 3c = 0 \right\}$$

Prove the two closure properties and **any one** of the other vector space properties.

11. (5 points) Show that the following set is a subspace of $M_{2\times 2}$.

$$L = \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \; : \; a, b, c \in \mathbb{R} \right\}$$

12. (3 points) Show that the following set is NOT a subspace of $M_{2\times 2}$.

$$U = \left\{ \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \; : \; a \in \mathbb{R} \right\}$$

13. (6 points) Is the vector $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$ in the span of this set? You may use your calculator as you see fit, but be sure to show work and/or explain your reasoning.

$$\left\{ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

14. (8 points) Consider the subspace of P_3 described below.

$$M = \{a + bx + cx^2 + dx^3 : a - 2b + c - d = 0\}$$

Find a set of vectors whose span is M. Equivalently, find a parameterization for the subspace.

15. (5 points) What does it mean to be a linear combination of x and y? After answering, prove that a linear combination of three linear combinations of x and y is a linear combination of x and y.