Math 236 - Assignment 1

January 22, 2025

Name ____________________

Score __________

Show all work to receive full credit. Supply explanations when necessary. Do all computations by hand. This assignment is due January 29.

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1. Find the solution set by reducing to echelon form.

$$\begin{array}{rcrcrcr} x_1 & - & 3x_2 & + & 4x_3 & = & -4 \\ 3x_1 & - & 7x_2 & + & 7x_3 & = & -8 \\ -4x_1 & + & 6x_2 & - & x_3 & = & 7 \end{array}$$

2. Find the solution set by reducing to echelon form.

$$\begin{array}{rcrcrcr} x_1 & & & - & 3x_3 & = & 8 \\ 2x_1 & + & 2x_2 & + & 9x_3 & = & 7 \\ & & x_2 & + & 5x_3 & = & -2 \end{array}$$

3. Prove that each elementary row operation in Gauss's method is reversible.

4. For which values of b are there no solutions, infinitely many solutions, or a unique solution?

$$\begin{array}{rcrcr} 2x & + & y & = & 7 \\ 8x & + & 4y & = & b \end{array}$$

5. Consider the system shown below with variables x and y. Use geometric reasoning to explain why there are three possibilities: no solution, infinitely many solutions, unique solution.

$$\begin{array}{rcrcr} ax & + & by & = & c \\ dx & + & ey & = & f \end{array}$$

6. Find the solution set by reducing to echelon form. Write the solution set in vector notation, identifying a particular solution and the solution of the corresponding homogeneous system.

$$\begin{array}{rcrcrcrcr} x & & & - & z & & & = & 1 \\ & & y & + & 2z & - & w & = & 3 \\ x & + & 2y & + & 3z & - & w & = & 7 \end{array}$$

7. Find the coefficients a, b, and c so that the graph of $p(x) = ax^2 + bx + c$ passes through the points $(1, -6)$, $(2, -9)$, and $(-1, -12)$.

8. Describe all functions $f(x) = ax^2 + bx + c$ such that $f(1) = 2$. Do so by writing $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \text{particular} + \text{homogeneous}$.