Math 236 - Assignment 1

January 21, 2026

Name ____________________

Score ____________

Show all work to receive full credit. Supply explanations when necessary. Do all computations by hand. This assignment is due January 28.

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1. Find the solution set of the homogeneous system by reducing to echelon form.

$$\begin{array}{rcrcrcrcrcl} x_1 & + & 2x_2 & + & x_3 & + & 4x_4 & + & x_5 & = & 0 \\ 2x_1 & + & 6x_2 & + & 3x_3 & + & 11x_4 & + & x_5 & = & 0 \\ x_1 & + & 4x_2 & + & 2x_3 & + & 7x_4 & & & = & 0 \end{array}$$

2. Find the solution set by reducing to echelon form.

$$\begin{array}{rcrcrcrcl} x_1 & + & 3x_2 & + & x_3 & + & 2x_4 & = & 1 \\ 2x_1 & + & 6x_2 & + & 4x_3 & + & 8x_4 & = & 3 \\ & & & & 2x_3 & + & 4x_4 & = & 1 \end{array}$$

3. Find the number b that makes the coefficient matrix singular. Then, with that b, find the right-hand side g that makes the system solvable. Finally, find the solution set for that singular case.

$$\begin{array}{rcrcl} 3x & + & 4y & = & 16 \\ 4x & + & by & = & g \end{array}$$

4. For constants a, b, c, and d, the equation $ax + by + cz = d$ describes a plane in 3-dimensional space. By reducing an appropriate 3×4 linear system to echelon form, find an equation of the plane that passes through the points $(1, 2, 3)$, $(-2, 1, 0)$, and $(3, -4, 1)$.

5. Describe all functions $g(x) = ax^2 + bx + c$ such that $g(1) = 4$ and $g(-1) = 2$. Do so by writing $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \text{particular} + \text{homogeneous}$.

6. Show that each statement is false by giving a counterexample.

   (a) A linear system has at most one particular solution.

   (b) A linear system has at least one particular solution.

   (c) Every underdetermined system (more unknowns than equations) has infinitely many solutions.

   (d) Every overdetermined system (more equations than unknowns) has no solution.

7. Use our definition of *nonsingular* to show that the following matrix is nonsingular.

$$\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$$

Turn over.

8. Use our definition of *nonsingular* to show that the following matrix is singular.

$$\begin{pmatrix} 1 & 2 & 1 \\ 4 & 1 & 0 \\ 3 & -1 & -1 \end{pmatrix}$$