MTH 236 Assignment #1 Key

1) $$\begin{aligned} x_1 + 2x_2 + x_3 + 4x_4 + x_5 &= 0 \\ 2x_1 + 6x_2 + 3x_3 + 11x_4 + x_5 &= 0 \\ x_1 + 4x_2 + 2x_3 + 7x_4 &= 0 \end{aligned}$$

$R_2 = -2R_1 + R_2$

$R_3 = -R_1 + R_3$

$$\begin{aligned} x_1 + 2x_2 + x_3 + 4x_4 + x_5 &= 0 \\ 2x_2 + x_3 + 3x_4 - x_5 &= 0 \\ 2x_2 + x_3 + 3x_4 - x_5 &= 0 \end{aligned}$$

$R_3 = -R_2 + R_3$

$$\begin{aligned} x_1 + 2x_2 + x_3 + 4x_4 + x_5 &= 0 \\ 2x_2 + x_3 + 3x_4 - x_5 &= 0 \\ 0 &= 0 \end{aligned}$$

Leading variables: x_1, x_2 Free variables: x_3, x_4, x_5

$$\left.\begin{aligned} x_5 &= x_5 \\ x_4 &= x_4 \\ x_3 &= x_3 \\ x_2 &= -\tfrac{1}{2}x_3 - \tfrac{3}{2}x_4 + \tfrac{1}{2}x_5 \\ x_1 &= -2x_2 - x_3 - 4x_4 - x_5 \\ &= (x_3 + 3x_4 - x_5) - x_3 - 4x_4 - x_5 \\ &= -x_4 - 2x_5 \end{aligned}\right\}$$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ -1/2 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} -1 \\ -3/2 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_4 + \begin{pmatrix} -2 \\ 1/2 \\ 0 \\ 0 \\ 1 \end{pmatrix} x_5; \quad x_3, x_4, x_5 \in \mathbb{R}}$$

2)

$$x_1 + 3x_2 + x_3 + 2x_4 = 1$$
$$2x_1 + 6x_2 + 4x_3 + 8x_4 = 3$$
$$2x_3 + 4x_4 = 1$$

$R_2 = -2R_1 + R_2$

$$x_1 + 3x_2 + x_3 + 2x_4 = 1$$
$$2x_3 + 4x_4 = 1$$
$$2x_3 + 4x_4 = 1$$

$R_3 = -R_2 + R_3$

$$x_1 + 3x_2 + x_3 + 2x_4 = 1$$
$$2x_3 + 4x_4 = 1$$
$$0 = 0$$

LEADING VARIABLES : x_1, x_3

FREE VARIABLES : x_2, x_4

$$\left.\begin{aligned} x_4 &= x_4 \\ x_2 &= x_2 \\ x_3 &= \tfrac{1}{2}(1 - 4x_4) \\ x_1 &= 1 - 3x_2 - x_3 - 2x_4 \\ &= 1 - 3x_2 - \tfrac{1}{2} + 2x_4 - 2x_4 \\ &= \tfrac{1}{2} - 3x_2 \end{aligned}\right\}$$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix} x_4; \quad x_2, x_4 \in \mathbb{R}}$$

3) $$3x + 4y = 16$$
$$4x + by = g$$

FIRST LET'S LOOK AT THE HOMOGENEOUS SYSTEM:

$$\begin{aligned} 3x + 4y &= 0 \\ 4x + by &= 0 \end{aligned} \xrightarrow{R_2 = -\frac{4}{3}R_1 + R_2} \begin{aligned} 3x + 4y &= 0 \\ \left(b - \tfrac{16}{3}\right)y &= 0 \end{aligned}$$

FOR THE COEFFICIENT MATRIX TO BE

SINGULAR, $b - \frac{16}{3} = 0$ OR $\boxed{b = \frac{16}{3}}$

NOW, WITH $b = \frac{16}{3}$,

$$\begin{aligned} 3x + 4y &= 16 \\ 4x + \tfrac{16}{3}y &= g \end{aligned} \xrightarrow{R_2 = -\frac{4}{3}R_1 + R_2} \begin{aligned} 3x + 4y &= 16 \\ 0 &= g - \tfrac{64}{3} \end{aligned}$$

IN ORDER TO BE

SOLVABLE

$$g - \frac{64}{3} = 0$$

OR

$$\boxed{g = \frac{64}{3}}$$

SOLUTION SET...

$$3x + 4y = 16 \Rightarrow y = y, \; x = \frac{1}{3}(16 - 4y)$$

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16/3 \\ 0 \end{pmatrix} + \begin{pmatrix} -4/3 \\ 1 \end{pmatrix} y, \; y \in \mathbb{R}}$$

4)

$$a + 2b + 3c - d = 0$$
$$-2a + b - d = 0$$
$$3a - 4b + c - d = 0$$

$R_2 = 2R_1 + R_2$

$R_3 = -3R_1 + R_3$

$$a + 2b + 3c - d = 0$$
$$5b + 6c - 3d = 0$$
$$-10b - 8c + 2d = 0$$

$R_3 = 2R_2 + R_3$

$$a + 2b + 3c - d = 0$$
$$5b + 6c - 3d = 0$$
$$4c - 4d = 0$$

LEADING VARIABLES a, b, c. FREE VARIABLE d.

$$d = d$$
$$c = d$$
$$5b = -3d \Rightarrow b = -\frac{3}{5}d$$
$$a = \frac{6}{5}d - 3d + d = -\frac{4}{5}d$$

CHOOSE $d = -5$.
PLANE IS GIVEN BY

$$4x + 3y - 5z = -5$$

5)

$g(x) = ax^2 + bx + c$

$g(1) = 4$

$g(-1) = 2$

$\Downarrow$

$a + b + c = 4$

$a - b + c = 2$

$R_2 = -R_1 + R_2 \longrightarrow$
$$\begin{aligned} a + b + c &= 4 \\ -2b \quad &= -2 \end{aligned}$$

$R_2 = -\frac{1}{2}R_2 \rightarrow$
$$\begin{aligned} a + b + c &= 4 \\ b \quad &= 1 \end{aligned}$$

$b = 1$

$a = 4 - b - c = 4 - 1 - c = 3 - c$

$c = c$ (FREE VARIABLE)

$$\boxed{\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} c, \quad c \in \mathbb{R}}$$

6)

(a). $x + y = 0$ HAS INFINITELY MANY PARTICULAR SOLUTIONS. ANY ORDERED PAIR $(x, -x)$ IS A PARTICULAR SOLUTION.

(b) THE SYSTEM
$$\begin{aligned} 2x + y &= 7 \\ 2x + y &= 5 \end{aligned}$$
HAS NO SOLUTIONS, WHICH AUTOMATICALLY MEANS NO PARTICULAR SOLUTION.

(c) THE SYSTEM
$$\begin{aligned} x + y + z &= 1 \\ x + y + z &= 2 \end{aligned}$$
HAS NO SOLUTION.

(d) THE SYSTEM
$$\begin{aligned} x + y &= 1 \\ 2x + 2y &= 2 \\ 3x + 3y &= 3 \end{aligned}$$
HAS INFINITELY MANY SOLUTIONS.

7) SHOW THAT $\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$ IS NONSINGULAR BY SHOWING THAT IT IS THE COEFFICIENT MATRIX OF A HOMOGENEOUS SYSTEM WITH A UNIQUE SOLUTION.

$$\begin{aligned} x + 2y &= 0 \\ -2x + 3y &= 0 \end{aligned} \quad \xrightarrow{R_2 = 2R_1 + R_2} \quad \begin{aligned} x + 2y &= 0 \\ 7y &= 0 \end{aligned}$$

$$7y = 0 \Rightarrow y = 0$$

$$x + 2(0) = 0$$

$$\Rightarrow x = 0$$

Homo. SYSTEM HAS THE SINGLE SOLUTION $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

8) SHOW THAT $\begin{pmatrix} 1 & 2 & 1 \\ 4 & 1 & 0 \\ 3 & -1 & -1 \end{pmatrix}$ IS SINGULAR BY SHOWING THAT IT IS THE COEFFICIENT MATRIX OF A HOMOGENEOUS SYSTEM WITH INFINITELY MANY SOLUTIONS.

$$\begin{aligned} x + 2y + z &= 0 \\ 4x + y \quad &= 0 \\ 3x - y - z &= 0 \end{aligned}$$

$R_2 = -4R_1 + R_2$
$R_3 = -3R_1 + R_3$

$$\begin{aligned} x + 2y + z &= 0 \\ -7y - 4z &= 0 \\ -7y - 4z &= 0 \end{aligned}$$

$R_3 = -R_2 + R_3$

$$\begin{aligned} x + 2y + z &= 0 \\ -7y - 4z &= 0 \end{aligned}$$

THE SYSTEM IS CONSISTENT ($x = y = z = 0$ IS A SOLUTION) AND THERE IS A FREE VARIABLE. THEREFORE THERE ARE INF. MANY SOLUTIONS.